APPLICATION OF RUNGE KUTTA METHODS IN INTUITIONISTIC TRIANGULAR FUZZY MAGDM PROBLEMS

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Abstract: In this paper, the existing multiple attribute group decision making focuses on models based on the application of numerical methods with Intuitionistic Fuzzy Triangular sets (ITrFs). In MAGDM problems the decision maker weights play a significant role. The unknown decision maker weights are derived from the ordinary differential equations. Initial value problems of Runge Kutta methods are applied in Decision Making Problems. For decision making process the Intuitionistic Triangular Fuzzy Weighted Geometric (ITrFWG) operator and the intuitionistic Triangular Fuzzy Hybrid Geometric (ITrFHG) operators are used. Euclidean distance measure is utilized as a tool to rank the best alternatives. Numerical examples are used to illustrate the feasibility and effectiveness of the proposed method.

Keywords: Intuitionistic Triangular Fuzzy sets (ITrFs), Intuitionistic Triangular Fuzzy Weighted Geometric (ITrFWG), Intuitionistic Triangular Fuzzy Hybrid Geometric (ITrFHG), Runge-Kutta method

I. INTRODUCTION

Atanassov, (1986,1989,1994) introduced the concept of Intuitionistic fuzzy sets. Atanassov, (1994) also developed the concept of Intuitionistic fuzzy sets. Li, (2005) defined some Multiple attribute decision making models and methods using intuitionistic fuzzy sets. Liu, & Wang, (2007) proposed a triangular fuzzy number using the method of multiple attribute decision making with attribute weight information. Arumugham, (2012) has solved various numerical methods and its applications. Iyengar & Jain also created the Numerical solution of differential equation problems. In this paper intuitionistic triangular weighted geometric and hybrid geometric operators are used to simplify the matrix into a single matrix. Xu, & Yager, (2006) has investigated geometric aggregation operators under intuitionistic fuzzy sets. Wei, Zhao, Lin, & Wang, (2012) applied the multiple attribute group decision making in generalized triangular fuzzy averaging operator. Wan, Wang, Lin, & Dong, (2016) also identified new generalized aggregation operators in intuitionistic triangular fuzzy sets and it is applied in multiple attribute decision making problems. Robinson & Amirtharaj, (2011, 2012, 2015) defined correlation coefficient for triangular intuitionistic fuzzy sets in MAGDM problems and also they extended trapezoidal intuitionistic fuzzy sets. Robinson, & Akila, (2019) determined some numerical method techniques for finding the attribute weight in MAGDM problems. In Numerical Analysis Runge-Kutta methods are explicit and implicit iterative methods which are used for finding the approximate ordinary differential equations. Runge kutta method is the generalization of the Runge Kutta 4th order method. In this paper Runge Kutta methods are used to obtain the solution of numerical methods and to fund the decision maker weights in MAGDM problems based on triangular intuitionistic fuzzy sets. Numerical illustrations are shown to prove the effectiveness of the proposed methods.

Definition: 1 Intuitionistic Fuzzy Set

Let a set X be fixed. An IFS Ã in X is an object having the form $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x) >, x \in X\}$, where the $\mu_{\tilde{A}}(x): X \to [0, 1]$ and $\gamma_{\tilde{A}}(x): E \to [0, 1]$ define the degree of membership and degree of non-membership respectively, of the element x∈X to the set Ã, which is a subset of X, for every element x ∈ X, $0 \le \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \le 1$.

Definition: 2 Intuitionistic Fuzzy Number

1. An IFN Ã is defined as follows:
2. An intuitionistic fuzzy sub set of the real line.
3. Normal i.e there is any $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$ (so $\gamma_{\tilde{A}}(x_0) = 0$).
4. Convex for the membership function $\mu_{\tilde{A}}(x)$
   $i.e \quad \mu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda)x_2\right) \geq min\{ \mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, $x_{1,} x_2 \in R$, $\lambda \in [0,1]$.
5. Concave for the non- membership function $\gamma_{\tilde{A}}(x)$
   $i.e \quad \gamma_{\tilde{A}}\left(\lambda x_1 + (1-\lambda)x_2\right) \leq max\{\gamma_{\tilde{A}}(x_1), \gamma_{\tilde{A}}(x_2)\}, x_{1,} x_2 \in R$, $\lambda \in [0,1]$.

Definition: 3 Triangular Fuzzy Number (TrFN) $A = (a,b,c)$ is called a triangular fuzzy number, if the membership function $\mu_A : R \to [0,1]$ is expressed as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{x-c}{b-c} & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

where $x \in R, \quad 0 \leq a \leq b \leq c \leq 1.$

II. SOME GEOMETRIC AGGREGATION OPERATORS WITH INTUITIONISTIC TRIANGULAR FUZZY NUMBERS

Defintion: 4 Let $\tilde{\alpha}_j\left(j = 1,2,...,n\right)$ be collection of intuitionistic triangular fuzzy numbers, and let $\text{ITrFWG}:\Omega^n \to \Omega$. Then $\text{ITrFWG}_\omega\left(\tilde{\alpha}_1, \tilde{\alpha}_1, ..., \tilde{\alpha}_n\right) = \tilde{\alpha}_1^{\omega_1} \otimes \tilde{\alpha}_1^{\omega_1} .... \otimes \tilde{\alpha}_1^{\omega_1}$, is called the Intuitionistic Triangular Fuzzy Weighted Geometric (ITrFWG) operator.

Theorem: 1 Let $\tilde{\alpha}_j\left(j = 1,2,...,n\right)$ be a collection of intuitionistic triangular fuzzy numbers. Then the aggregated value by using the ITrFWG operator is also an intuitionistic triangular fuzzy number and

$$\text{ITrFWG}_\omega\left(\tilde{\alpha}_1, \tilde{\alpha}_1, ..., \tilde{\alpha}_n\right) = \prod_{j=1}^{n} \tilde{\alpha}_j^{\omega_j} = \left(\left[\prod_{j=1}^{n} \alpha_j^{\omega_j}, \prod_{j=1}^{n} b_j^{\omega_j}, \prod_{j=1}^{n} c_j^{\omega_j}\right]; \prod_{j=1}^{n} \mu_{\tilde{\alpha}_j}^{\omega_j}, 1 - \prod_{j=1}^{n}\left(1-\gamma_{\tilde{\alpha}_j}\right)^{\omega_j}\right),$$

where $\omega = \left(\omega_1, \omega_2, ..., \omega_n\right)^T$ is the weight vector of $\tilde{\alpha}_j\left(j = 1,2,...,n\right)$, with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1.$

Definition

Let $\tilde{\alpha}_j$ (j = 1,2, . . . n) be a collection of intuitionistic triangular fuzzy numbers. An intuitionistic triangular fuzzy Hybrid Geometric operator of dimension n is a mapping ITrHG $:\Omega^n \to \Omega$ that has an associated vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ such that $\omega_j \epsilon [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$

ITrFWG_w $(\widetilde{\alpha_1}$, $\widetilde{\alpha_2}$,. . . , $\widetilde{\alpha_n})$ = $\widetilde{\alpha_{\sigma(J)}^{\omega_J}} \otimes \widetilde{\alpha_{\sigma(J)}^{\omega_J}} \otimes$, . . . , $\otimes \widetilde{\alpha_{\sigma(J)}^{\omega_J}}$ where $\widetilde{\alpha_{\sigma(J)}^{\omega_J}}$ is the jth largest of the weighted intuitionistic triangular fuzzy numbers . ITrFWG_w =

$\widetilde{\alpha_1}$, $\widetilde{\alpha_2}$,. . . , $\widetilde{\alpha_n}$) =($\prod_{j=1}^{n} \widetilde{\alpha_{\sigma(J)}^{\omega_J}}$, $\prod_{j=1}^{n} \widetilde{b_{\sigma(J)}^{\omega_J}}$, $\prod_{j=1}^{n} \widetilde{c_{\sigma(J)}^{\omega_J}}$) , $\prod_{j=1}^{n} \widetilde{\mu_{\sigma(J)}^{\omega_J}}$, 1- $\prod_{j=1}^{n}(1 - \widetilde{\gamma_J})^{\omega_J}$)

III. ALGORITHM FOR MAGDM PROBLEMS WITH INTUITIONISTIC TRIANGULAR FUZZY INFORMATION

Step1: Use the decision information given by the intuitionistic triangular fuzzy decision matrix $\widetilde{R_k}$ and the ITrFWG operator $r_i^{\sim(k)} = \left(\left[a^{(k)}, b^{(k)}, c^{(k)}\right]; \mu^{(k)}, \gamma^{(k)}\right) = ITrFWG_w\left(r_{i1}^{\sim(k)}, r_{i2}^{\sim(k)}, \ldots, r_{in}^{\sim(k)}\right)$ here i = 1, 2, …, m; k = 1,2,…, t to derive the individual overall interval valued intuitionistic triangular fuzzy numbers $r_i^{\sim(k)}$ of the alternative A_i.

Step2: Use the ITrFHG operator to derive the collective overall intuitionistic triangular fuzzy values $\widetilde{r_i}$(i = 1,2,…m) of the Alternative A_i . $(r_i = [a_i, b_i, c_i, \mu_i, \gamma_i]) = ITrFHG_w(r_i^1, r_i^2, \ldots r_i^i)$ i =1, 2, … m. where v = (v_1, v_2, …v_n) is the weighting vector of decision makers , with $v_i \in [0,1], \sum_{j=1}^{n} w_i = 1$.

Step3: The distance between the collective overall values $\widehat{r_i} = [a_i, b_i, c_i, \mu_i, \gamma_i]$ and triangular intuitionistic fuzzy positive ideal value $r^+ = [a^+, b^+, c^+, \mu^+, \gamma^+]$ = [1,1,1,1,0] using the distance formula

Step4:

Rank alternative A_i (i = 1,2,…n) and select in accordance with distance formula.

IV. NUMERICAL ILLUSTRATION

In this section a numerical example are considered to investigate the performance of the proposed method

Five candidates are challenging for the Chairperson in a college the masses satisfaction with the candidates should be evaluated .The final five campaigners of the college will be assessed. They are denoted as four alternatives namely A_1, A_2, A_3, A_4, A_5. The masses consist of four teams.

E_1: Under Graduate students

E_2: Post Graduate Students

E_3: Research Scholar

E_4: Administrative officers (Principal, Vice Principal, Dean Academics, Dean Student Affairs,

Dean Research). They are considered decision makers for the assessment of grade.

The following four attributes are used by decision makers, E_1, E_2, E_3, E_4 to evaluate the alternatives. Four attributes are denoted as follows.

A_1: Academic performance

A_2: Co-Curricular Activities

A_3: Personality

A_4: Discipline

A_5: Attendance

Decision makers E_1, E_2, E_3, E_4 provide decision matrices R_1, R_2, R_3 respectively. These decision matrices are expressed by intuiionistic triangular fuzzy numbers by the three decision makers whose weighting vectors are W =

(0.3034552, 0.2720597, 0.2320504, 0.1924348)^T under the above four attributes Runge Kutta fourth order weighting vector $\gamma = (0.3124124, 0.3320806, 0.3555070)$ $W = (0.3135535, 0.3320875, 0.3543591)$ respectively, the decision matrices as listed in the following matrices R = $r_{2ij\ 5x4}^{(k)}$ (k = 1,2,3) as follows. .

$$R_1 = \begin{pmatrix} [0.4,0.5,0.6],0.4,0.3) & [0.5,0.6,0.7],0.3,0.3) & [0.7,0.8,0.6],0.5,0.4) & [0.8,0.7,0.6],0.1,0.3) \\ [0.3,0.5,0.7],0.6,0.2) & [0.2,0.4,0.6],0.7,0.3) & [0.5,0.5,0.7],0.4,0.5) & [0.2,0.5,0.7],0.4,0.3) \\ [0.6,0.7,0.8],0.5,0.3) & [0.6,0.7,0.4],0.5,0.2) & [0.1,0.5,0.9],0.2,0.6) & [0.6,0.4,0.3],0.2,0.7) \\ [0.2,0.4,0.6],0.6,0.3) & [0.1,0.4,0.5],0.3,0.6) & [0.5,0.6,0.7],0.4,0.3) & [0.9,0.7,0.5],0.6,0.3) \\ [0.7,0.8,0.9],0.4,0.5) & [0.4,0.7,0.8],0.6,0.4) & [0.6,0.7,0.8],0.5,0.2) & [0.5,0.4,0.3],0.5,0.3) \end{pmatrix}$$

$$R_2 = \begin{pmatrix} [0.5,0.6,0.7],0.3,0.5) & [0.4,0.3,0.5],0.6,0.2) & [0.4,0.7,0.6],0.7,0.2) & [0.4,0.5,0.6],0.3,0.4) \\ [0.1,0.2,0.3],0.2,0.6) & [0.6,0.2,0.6],0.5,0.4) & [0.6,0.6,0.4],0.5,0.3) & [0.7,0.5,0.4],0.7,0.3) \\ [0.6,0.7,0.4],0.3,0.3) & [0.3,0.5,0.4],0.4,0.4) & [0.7,0.6,0.5],0.4,0.3) & [0.1,0.5,0.3],0.4,0.4) \\ [0.6,0.4,0.2],0.8,0.2) & [0.6,0.6,0.3],0.2,0.3) & [0.2,0.5,0.7],0.3,0.5) & [0.6,0.7,0.9],0.6,0.2) \\ [0.5,0.4,0.4],0.3,0.4) & [0.5,0.6,0.7],0.4,0.3) & [0.4,0.3,0.2],0.2,0.6) & [0.3,0.5,0.7],0.3,0.2) \end{pmatrix}$$

$$R_3 = \begin{pmatrix} [0.6,0.7,0.8],0.2,0.5) & [0.5,0.4,0.3],0.4,0.4) & [0.7,0.8,0.9],0.6,0.3) & [0.6,0.5,0.4],0.4,0.3) \\ [0.4,0.2,0.6],0.3,0.4) & [0.5,0.3,0.5],0.4,0.3) & [0.5,0.7,0.4],0.1,0.3) & [0.7,0.5,0.3],0.3,0.6) \\ [0.2,0.4,0.6],0.4,0.4) & [0.8,0.7,0.4],0.2,0.7) & [0.6,0.4,0.2],0.4,0.4) & [0.4,0.3,0.5],0.1,0.5) \\ [0.7,0.8,0.9],0.3,0.6) & [0.6,0.7,0.8],0.5,0.3) & [0.6,0.4,0.3],0.2,0.5) & [0.5,0.6,0.7],0.8,0.1) \\ [0.5,0.7,0.9],0.6,0.3) & [0.9,0.8,0.7],0.4,0.2) & [0.9,0.8,0.7],0.7,0.2) & [0.1,0.3,0.5],0.2,0.4) \end{pmatrix}$$

Runge Kutta Method of Fourth order:

The initial value problem is given by $\frac{dy}{dx}$ = f(x,y), y(x_j , y), y(x_i) $= y_i$

k_1 = hf(x_i , y_i)

k_2 = hf($x_i + \frac{h}{2}$, $y_i + \frac{k_1}{2}$)

k_3 = hf($x_i + \frac{h}{2}$, $y_i + \frac{k_2}{2}$)

k_4 = hf($x_i +\ h$, $y_i +\ k_3$)

$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

Hence the required approximate value is given by

$y_i = y_0 + \Delta y$.

$u_{j+1=}\ u_j + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ for j = 0,1,2

k_1 = hf(t_j , u_j)

k_2 = hf($t_j + \frac{h}{2}$, $u_j + \frac{k_1}{2}$)

In Runge- Kutta third order Method:

k_1 = hf(t_j , u_j)

k_2 = hf($x_i + \frac{h}{2}, y_i + \frac{k_1}{2}$)

k_3 = hf($x_i + h , y_i - k_1 + 2k_2$)

$y_{j+1=} y_j + \frac{1}{6}(k_1 + 4k_2 + k_3)$

Problem proposed by decision maker1:

For the initial value problem u' = -2tu^2, u(0) = 1 with h = 0.2 on the interval [0,0.6] use the fourth order classical Runge Kutta Method.

Solution:

Given u' = -2tu^2, u(0) = 1 with h = 0.2

For j = 0 t_0= 0 u_0= 1

k_1 = 0.2 (-2) f(0, 1) = 0

k_2 = (0.2)-2f(0+ $\frac{0.2}{2}, 1 + \frac{0}{2}$) = -0.04

k_3 = (0.2)-2f() = -0.038416

k_4 = -0.0739715

The approximate value of y_1 = y(0.2)= 0.9615328

Similarly, we obtain

Y_2 = y(0.4)= 0.8620525

Y_3 = y(0.6)= 0.7352784

Y_4 = y(0.8)= 0.6097519

Normalization of y = ($\frac{y_1}{y} + \frac{y_2}{y} + \frac{y_3}{y} + \frac{y_4}{y}$)

Normalization of y ≈ 1

The weighting vectors are W = (0.3034552, 0.2720597, 0.2320504, 0.1924348)

Problem proposed by Decision maker 2

Solve the initial value problem using second and third order Runge Kutta method $\frac{dy}{dx}$ = (1+xy) , y(0) = 2 with h = 0.1

Runge Kutta second order method:

$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$

$where\ k_1$ = hf(x_0 , y_0)

k_2 = hf($x_0 + h$, $y_0 + k_1$)

k_1 = (0.1)(1+0) = 0.1

k_2 = (0.1)[1+(0.1)(2.1)] = 0.121

y_1 = 2+$\frac{1}{2}$(0.1+0.121) = 2.1105

Similarly

Y(0.2) = 2.2433686

Y(0.3) = 2.4016258

The weighting vectors are $\gamma =$ (0.3124124, 0.3320806, 0.3555070)

Runge Kutta third order method

k_1 = (0.1)(1+0) = 0.1

k_2 = 0.11025

k_3 = (0.1)[1+(0.1)(2-(0.1)+2(0.11025))] = 1.21205

y_1 = 2+$\frac{1}{6}$(0.1+ 4(0.11025)+1.21205 = 2.292175

Similarly

Y(0.2) = 2.4276645

Y(0.3) = 2.5904768

The weighting vectors are $W =$ (0.3135535, 0.3320875, 0.3543591)

Step1:

By using ITrFWG operator

$r_1^{(1)} =$ $((0.4)^{0.3034552}*(0.5)^{0.2720597}*(0.7)^{0.23205045}*(0.8)^{0.1924348}),((0.5)^{0.3034552}*(0.6)^{0.2720597}*(0.8)^{0.23205045}*(0.7)^{0.1924348}),((0.6)^{0.3034552}*(0.7)^{0.2720597}*(0.6)^{0.23205045}*(0.6)^{0.1924348})$, $((0.4)^{0.3034552}*(0.3)^{0.2720597}*(0.5)^{0.23205045}*(0.1)^{0.1924348})$, $(1- ((1-0.3)^{0.3034552}*(1-0.3)^{0.2720597}*(1-0.4)^{0.23205045}*(1-0.3)^{0.1924348}))$

$r_1^{(1)}$=(0.553032910146741,0.62517015690003,0.625697962283991,0.298332010259212, 0.324597001913700)

Similarly

$r_2^{(1)}$= (0.279774325341438, 0.470548778226905, 0.671250300727108, 0.526764620589006, 0.325802871899199)

$r_3^{(1)}$= (0.395895777097405, 0.581326287379252, 0.563755251169382, 0.338883399540838, 0.458414958330190)

$r_4^{(1)}$ = (0.273636948092575, 0.489430427765954, 0.571356268463257, 0.452262970499870, 0.398857880579832)

$r_5^{(1)}$ = (0.543653271986917, 0.654526140871866 , 0.686501141302134, 0.491025603228012 , 0.374829843099962)

$r_1^{(2)}$ = (.279774325341438 , 0.470548778226905. 0.671250300727108 , 0.526764620589006, 0.325802871899199)

$r_2^{(2)}$= (0.358839353750249 , 0.307838970999122 , 0.409311510089642, 0.403955116780262 ,

0.433586842033655)

$r_3^{(2)}$ =(0.364791524300919 , 0.577679977738676 , 0.398570712468946, 0.366561308668059 , 0.348369203442160)

$r_4^{(2)}$ = (0.464980794522076 , 0.523870878595596 , 0.398923186939495, 0.413432786954618) , 0.308254142710082)

$r_5^{(2)}$ = (0.430319005206215 , 0.436138108426201 , 0.441667682658474, 0.295290799434343, 0.398071030441381)

$r_1^{(3)}$ = (0.428023662666335, 0.497222636669533 , 0.598307631681397, 0.440969431997557 , 0.343694352299338)

$r_2^{(3)}$=(0.498519592240185 , 0.356259100000426 , 0.454797939221099, 0.251417653995649 ,

0.400190020671343)

$r_3^{(3)}$= (0.430002519577458 , 0.440695107870026 , 0.402059615401045, 0.253690676934004 ,

0.520249032157046)

$r_4^{(3)}$ = (0.607056818218769, 0.621464025753965 , 0.643585967968667, 0.378953944927168 , 0.426622066837287)

$r_5^{(3)}$ = (0.493343336942320 , 0.636093278951398 , 0.708106130346734 , 0.450779524826352 , 0.273142202461787)

Step2:

Utilize the ITrFHG operator to derive the collective overall preference intuitionistic fuzzy values $\tilde{r_1}$ of the Alternative A_i Then we have

$r_1^{(1)} =$ (0.553032910146741, 0.625170156900038, 0.625697962283991, 0.298332010259212, 0.324597001913700)

$r_1^{(2)}$= 0.279774325341438, 0.470548778226905. 0.671250300727108, 0.526764620589006, 0.325802871899199

$r_1^{(3)}$= 0.428023662666335, 0.497222636669533, 0.598307631681397, 0.440969431997557, 0.343694352299338,

Where

γ = (0.3124124, 0.3320806, 0.3555070)^T

W = (0.3135535, 0.3320875, 0.3543591)^T

r_1 = (0.396810478126112, 0.047918466869450, 0.083188866662181, 0.022239112568973, 0.012126937011154)

r_2 = (0.369759648637586 , 0.016760333088120 , 0.040420221206569, 0.017058259448126 , 0.019907256582443)

r_3= (0.395651374876543 , 0.048500844148152 , 0.029491534003020 , 0.010267745000084 , 0.030777386485937)

r4 = (0.427129047211403 , 0.543206405420603 , 0.524128087004971, 0.409783594034616 , 0.378017977299587)

r_5 = (0.483343842779071 , 0.561170061493955 , 0.593759066223263, 0.397785449794035 , 0.350125010613406)

$a_1 = b^{nx\gamma_1}$ = (0.553032910146741)$^{3*0.3124124}$ = 0.573979848515193

$a_2 = b^{nx\gamma_2}$ = (0.279774325341438)$^{(3x0.3320806)}$= 0.281116840985570

$a_3 = b^{nx\gamma_3}$ = (0.428023662666335)$^{(3x0.3555070)}$ = 0.404531781582667

b_1= $b^{nx\gamma_1}$ = (0.625170156900038)$^{(3x0.3124124)}$ = 0.076334871405615

b_2 = $b^{nx\gamma_2}$ (0.470548778226905)$^{(3x0.3320806)}$ = 0.034598514722890

b_3 = $b^{nx\gamma_3}$ = (0.497222636669533)$^{(3x0.3555070)}$ = 0.043701951741137

C_1= $b^{nx\gamma_1}$ = (0.625697962283991)$^{(3x0.3124124)}$ = 0.076528373832373

C2 = $b^{nx\gamma_2}$ = (0.671250300727108)$^{(3x0.3320806)}$ = 0.100437752258463

C3= $b^{nx\gamma_3}$= (0.598307631681397)$^{(3x0.3555070)}$ = 0.076141562370676

μ_1= $b^{nx\gamma_1}$= (0.298332010259212)$^{(3*0.3124124)}$ = 0.008295218438640

μ_2= $b^{nx\gamma_2}$ = (0.526764620589006)$^{(3x0.3320806)}$ = 0.048539276703559

μ_3= $b^{nx\gamma_3}$ = (0.440969431997557)$^{(3x0.3555070)}$ = 0.030484116461731

γ_1= $b^{nx\gamma_1}$= (0.324597001913700)$^{(3*0.3124124)}$ = 0.010684686298664

γ_2= $b^{nx\gamma_2}$ = (0.325802871899199)$^{(3x0.3320806)}$ = 0.011484397863902

γ_3= $b^{nx\gamma_3}$ = (0.343694352299338)$^{(3x0.3555070)}$ = 0.014433290196601

r_1 = ((0.573979848515193)$^{0.3135535}$*(0.404531781582667)$^{0.3320875}$ *(0.281116840985570)$^{0.3543591}$), ((0.076334871405615)$^{0.3135535}$ *(0.043701951741137)$^{0.3320875}$* (0.034598514722890)$^{0.3543591}$),((0.100437752258463)$^{0.3135535}$*(0.076528373832373)$^{0.3320875}$*(0.076141562370676)$^{0.3543591}$) , ((0.048539276703559)$^{0.3135535}$ *(0.030484116461731)$^{0.3320875}$*(0.008295218438640)$^{0.3543591}$),(1-((1-0.014433290196601)$^{0.3135535}$*(1-0.011484397863902)$^{0.3320875}$*

(1- 0.010684686298664) $^{0.3543591}$))

r_1 = (0.396810478126112 , 0.047918466869450 , 0.083188866662181, 0.022239112568973 , 0.012126937011154)

Similarly

r_2 = (0.369759648637586 , 0.016760333088120 , 0.040420221206569, 0.017058259448126 , 0.019907256582443)

r_3= (0.395651374876543 , 0.048500844148152 , 0.029491534003020 , 0.010267745000084 , 0.030777386485937)

r_4 = (0.427129047211403 , 0.543206405420603 , 0.524128087004971, 0.409783594034616 , 0.378017977299587)

r_5 = (0.483343842779071 , 0.561170061493955 , 0.593759066223263, 0.397785449794035 , 0.350125010613406)

Step3:

The distance between the collective overall values $\widehat{r_i}$ = $[a_i, b_i, c_i, \mu_i, \gamma_i]$ and triangular intitionistic fuzzy positive ideal value r^+ = $[a^+ , b^+, c^+, \mu^+, \gamma^+]$ = [1,1,1,1,0] using the distance formula

d($\widetilde{r_1}, r^+$) = 0.693348449360743

d($\widetilde{r_2}, r^+$) = 0.618228918759369

d($\widetilde{r_3}, r^+$) = 0.695440852432285

d($\widetilde{r_4}, r^+$)= 0.557257988042305

d($\widetilde{r_5}, r^+$) = 0.535455777529013

Step 4:

Rank all the alternatives A_i $(I = 1,2,3,4,5)$

$A_5 < A_4 < A_3 < A_1 < A_2$

Hence the best alternatives is A_2

V. CONCLUSION

In this paper Runge kutta second, third and fourth order are used to obtain the solution of Numerical Methods and it is utilized to derive the Multiple Attribute Group Decision Making problems under Intuitionistic Triangular Fuzzy sets. In the process of determining weights multicriteria are explicitly considered the numerical solutions are decomposed and the decision makers weights for attributes and corresponding decision making methods have also been proposed. Finally an illustrative example was given to show the effectiveness of proposed method.

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