COMPUTATIONAL APPROACH FOR TRANSIENT BEHAVIOUR OF M/M(a,b)/1 BULK SERVICE QUEUEING SYSTEM WITH MULTIPLE VACATION AND DISCOURAGED CONSUMERS

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ABSTRACT–A new computational technique is used to evaluate the Transient behaviour of Single Server Bulk Service Queueing System with Multiple Vacation and Discouraged Consumers with arrival rate λ which follows a Poisson process and the service will be in bulk and the service time follows an exponential distribution with parameter μ and the vacation time follows an exponential distribution with parameter α. According to Neuts, the server begins service only when a minimum of 'a' consumers in the waiting room and a maximum service capacity is 'b'. An intensity matrix is formed for all transitions. Time dependent solutions and Steady state solutions are acquired by using Cayley Hamilton theorem. Numerical studies have been done for Time dependent average number of consumers in the queue, Transient probabilities of server in vacation and server busy for several values of t, λ, μ, α, p, a and b.

Keywords– **Single server, Bulk Service, Multiple vacation, Discouraged consumers, Infinitesimal matrix, Direct Truncation method, Cayley Hamilton, Exponential of a matrix, Time dependent solutions.**

I. INTRODUCTION

The main objective of this research paper is to analyze the transient behaviour of Bulk service queueing system by new computational approach. Bulk Queues bulk queue can be defined where consumers arrive in a collection of batches. This collection may be in terms of a random variable or fixed. In our case consumers arrive singly but are served in batches. Bulk service queues have potential applications in many areas e.g. in traffic signal systems, in computer networks where jobs are processed in batches, manufacturing/production systems, in restaurants, cinema halls, in transportation processes involving buses, airplanes, trains, ships, elevators, and so on.

In the study of queueing systems, determination of transient solution is very much essential to analyze the behavior of the system. Transient analysis is very useful for all queueing models to obtain optimal solutions which pave way to control the system. Even in the case of a simple M/M/1 queue, analytical approach to obtain transient behavior is very difficult .M.F. Neuts (1967) explained about general class of bulk queues with Poisson input.M.F. Neuts (1981) discussed about Matrix Geometric Solutions in Stochastic Models.

Rakesh Kumar (2017) explained transient solution to the M/M/c queueing model equations with balking and catastrophes. Sherif Ammar (2017) studied Transient solution of M/M/1 vacation queue with a waiting server and impatient consumers. Parthasarathy and Selvaraju (2001) have analysed transient analysis of a queue where potential consumers are discouraged by queue length. Sudhesh (2010) discussed the Transient analysis of queue with system disaster and consumers impatience.

R.Vimala Devi (2014) studied Bulk Queueing System with Multiple Vacations Set Up Times with N-Policy and Delayed Service. Kalidass and Ramanath, (2014) have analysed Transient behaviour of an M/M/1 queue with Multiple vacations. A.Senthil Vadivu & R. Arumuganathan (2015) have analysed Cost Analysis of MAP/ G (a, b)/1/N Queue with Multiple Vacations and Closedown Times. Sherif Ammar, (2015) explained about Transient analysis of an M/M/1 queue with impatient behaviour and Multiple vacation.

II. THE MATHEMATICAL MODEL AND ITS SOLUTIONS

A new computational method is used to estimate the Transient behaviour of Single server Bulk service queueing system with Multiple Vacation and Discouraged Consumers arrival rate λ succeeds a Poisson process and the service will be in bulk and the service time succeeds an exponential distribution with parameter μ and the vacation follows an exponential distribution with parameter α. Neuts had discussed general bulk service rule. The server begins service only when a minimum of 'a' consumers in the buffer (waiting room) and a maximum service capacity is 'b'. Assuming that there are 'a' consumers in the system at time t = 0. The general considerations for bulk service with multiple vacations and discouraged consumers are:

After completion of every service if the number of consumers in the queue is less than 'a', the server goes for vacation. After completion of the vacation period the server comes back to the system and starts service only if there are a minimum of 'a' consumers in the queue, if the server finds less than 'a' consumers waiting then he goes for vacation again, if he finds the number of consumers in the queue are between 'a' and 'b' then all the consumers in the queue will be taken for service and queue becomes empty and server starts service. If he finds more than 'b' consumers are waiting in the queue then the first 'b' consumers are taken for service and the remaining consumers will have to wait for service.

If the server is in vacation then the arriving customer may or may not join the system due to impatience. We assume that p is the probability that the customer joins the system and (1-p) is the probability that he leaves the system without getting service.

III. DESCRIPTION OF RANDOM PROCESS

Let N(t) be the random variable which represents the number of consumers in the queue at time t and C(t) be the random variable which represents the server status at time t. The random process is described as

$\{ < N(t), C(t) > / N(t) = 0,1,2,3,\ldots ; C(t) = 1,2\}$

C (t) = 1if the server is busy at time t

C (t) = 2 if the server is in vacation at time t

We define,

$P_{n1}(t)$: Probability that there are n consumers in the queue when the server is busy at time t

$P_{n2}(t)$: Probability that the server is in vacation when there are n consumers in the queue at time t

The Chapman-Kolmogorov equations are

$$P_{01}'(t) = -(\lambda+\mu)P_{01}(t)+\mu\sum_{n=a}^{b} P_{n1}(t)+\alpha\sum_{n=a}^{b} P_{n2}(t) \quad (1)$$

$$P_{02}'(t) = -\lambda p P_{02}(t)+\mu P_{01}(t) \quad (2)$$

$$\left.\begin{aligned}
P_{11}'(t) &= -(\lambda+\mu)P_{11}(t)+\lambda P_{01}(t)+\mu P_{b+11}(t)+\alpha P_{b+12}(t)\\
P_{12}'(t) &= -\lambda p P_{12}(t)+\lambda p P_{02}(t)+\mu P_{11}(t)\\
P_{21}'(t) &= -(\lambda+\mu)P_{21}(t)+\lambda P_{11}(t)+\mu P_{b+21}(t)+\alpha P_{b+22}(t)\\
P_{22}'(t) &= -\lambda p P_{22}(t)+\lambda p P_{12}(t)+\mu P_{21}(t)\\
&.\\
&.\\
&.\\
P_{a-11}'(t) &= -(\lambda+\mu)P_{a-11}(t)+\lambda P_{a-21}(t)+\mu P_{b+a-11}(t)+\alpha P_{b+a-12}(t)\\
P_{a-12}'(t) &= -\lambda p P_{a-12}(t)+\lambda p P_{a-22}(t)+\mu P_{a-11}(t)\\
P_{a1}'(t) &= -(\lambda+\mu)P_{a1}(t)+\lambda P_{a-11}(t)+\mu P_{b+a1}(t)+\alpha P_{b+a2}(t)\\
P_{a2}'(t) &= -(\lambda p+\alpha)P_{a2}(t)+\lambda p P_{a-12}(t)\\
&.\\
&.\\
&.
\end{aligned}\right\} \quad (3)$$

The intensity matrix **Q** for this model is given below

$$Q=\begin{pmatrix} A_{00} & A_{01} & A_{02} & A_{03} & A_{04} & ..... \\ A_{10} & A_{11} & A_{12} & A_{13} & A_{14} & ..... \\ A_{20} & A_{21} & A_{22} & A_{23} & A_{24} & ..... \\ A_{30} & A_{31} & A_{32} & A_{33} & A_{34} & ..... \\ . & . & . & . & . & ..... \\ . & . & . & . & . & ..... \end{pmatrix}$$

The matrices $A_{00,}A_{01},A_{10},A_{11},A_{20},A_{02},...$ are described in the Intensity matrix **Q** can be obtained from the following infinitesimal transition rates of process **X** as succeeds

$$\text{where } A_{ii}=\begin{cases}\begin{bmatrix}-(\lambda+\mu) & \mu \\ 0 & -\lambda p\end{bmatrix}, & i=0,1,2,...(a-1) \\ \begin{bmatrix}-(\lambda+\mu) & 0 \\ 0 & -(\lambda p+\alpha)\end{bmatrix}, & i=a,a+1,a+2,...\end{cases}$$

$$A_{ii+1}=\left\{\begin{bmatrix}\lambda & 0 \\ 0 & \lambda p\end{bmatrix},\right. \quad i=0,1,2,...$$

$$A_{i0}=\left\{\begin{bmatrix}\mu & 0 \\ \alpha & 0\end{bmatrix},\right. \; i=a \; to \, b$$

$$A_{i\,i-b}=\begin{bmatrix}\mu & 0 \\ \alpha & 0\end{bmatrix} \; i=b,b+1,...$$

Remaining all other entries are zero.
Further, we can write the above equations (1), (2) and (3) as

$X'(t)=AX(t)$ where $A=Q^T$

Where $[X(t)]^T=\left(P_{01}(t) \quad P_{02}(t) \quad P_{11}(t) \quad P_{12}(t) \quad . \quad . \quad . \quad . \quad .\right)$

Then we solve the above equations, we get

$$X(t)=e^{tA} \; X_0$$

When t=0, $X_0=X(0)=\left(1 \quad 0 \quad 0 \quad . \quad . \quad .\right)^T$

IV. DESCRIPTION OF COMPUTATIONAL METHOD

The following effective computational procedure is used to find the Time dependent probabilities of number of consumers in the queue at time t. The time dependent probabilities vector is denoted by

$X(t)=(P_{01}(t),P_{02}(t),P_{11}(t),P_{12}(t),P_{21}(t),...,P_{a-11}(t),P_{a-12}(t),P_{a1}(t),P_{a2}(t),...,P_{M1}(t),P_{M2}(t))^T$

Step 1: Assume that the matrix Q is finite that is the number of consumers in the queue at time t is M (sufficiently large). This value of M C can be taken such that the loss probability is small. And due to the nature of the system we can study about M only through the algorithmic methods . While a number of approaches are available for determining the cut-off point, M, the one that seems to perform well is to increase M until the largest individual change in the elements of **X (t)** for successive values is less than ε a predetermined infinitesimal value.

Step 2: Find the Eigen values of this finite order matrix tA.

Step 3: Let $d_1,d_2,d_3,...d_{2M}$ be 2M Eigen values.

Step 4: Use these Eigen values in the Vandermonde's matrix

$$V=\begin{pmatrix}1 & d_1 & d_1^2 & . & ... & d_1^{2M-1}\\ 1 & d_2 & d_2^2 & . & ... & d_2^{2M-1}\\ 1 & d_3 & d_3^2 & . & ... & d_3^{2M-1}\\ . & . & . & . & ... & .\\ . & . & . & . & ... & .\\ 1 & d_{2M} & d_{2M}^2 & . & ... & d_{2M}^{2M-1}\end{pmatrix}.$$

Step 5: Let $C=\left(e^{d_1} \quad e^{d_2} \quad . \quad . \quad . \quad e^{d_{2M}}\right)^T$ and $\alpha=\left(\alpha_0 \quad \alpha_1 \quad . \quad . \quad . \quad \alpha_{2M-1}\right)^T$.

Step 6: Find $\alpha=V^{-1}C$ and we get $\alpha=\left(\alpha_0 \quad \alpha_1 \quad \alpha_2 \quad . \quad . \quad . \quad \alpha_{2M-1}\right)^T$.

Step 7: Using α in $e^{tA}=\alpha_0 I+\alpha_1(tA)+\alpha_2(tA)^2+...+\alpha_{2M-1}(tA)^{2M-1}$

Step 8: Extract the first column of this Exponential matrix tA and store in X (t).

Step 9: This probability vector **X (t)** provides time dependent probabilities of number of consumers in the queue at time t.

V. SYSTEM PERFORMANCE MEASURES

The following system measures are used to bring out the Transient behaviour of bulk service queueing model under study. Numerical study has been dealt in very large scale to study the following measures for several values of t, λ, μ, α, p, a and b.

a. Probability that there are n consumers in the queue when the server is busy at time t = $P_{n1}(t)$

b. Probability that there are n consumers in the queue when the server is in vacation at time t = $P_{n2}(t)$

c. Probability that the server is busy at time t = $P_{busy}(t)=\sum_{n=0}^{\infty}P_{n1}(t)$

d. Probability that the server is in vacation at time t = $P_{vacation}(t)=\sum_{n=0}^{\infty}P_{n2}(t)$

e. Average number of consumers in the queue = $L_q(t)=\sum_{n=0}^{\infty}nP_{n1}(t)+\sum_{n=0}^{\infty}nP_{n2}(t)$

VI. NUMERICAL COMPUTATIONS

The Time dependent System performance measures and Transient probabilities of this model have been done and expressed in the form of tables, which are shown below for several values of t, λ, μ, α, p, a and b.

Table 1, Table 4, Table 7and Table 10 show Transient probabilities of number of consumers in the queue when the server is busy for several values of t, λ, μ, α, p, a and b.

We infer the following

- As the value of t increases the Transient Probabilities $P_{n1}(t)\to P_{n1}$
- .The sequence $\{P_{n1}(t)\}\to 0$ as n $\to\infty$ for all values of t

Table 2, Table 5, Table 8 and Table 11 show Transient probabilities of number of consumers in the queue when the server is in vacation for several values of t, λ, μ, α, p, a and b.

We infer the following

- As the value of t increases the Transient Probabilities $P_{n2}(t)\to P_{n2}$
- The sequence $\{P_{n2}(t)\}\to 0$ as n $\to\infty$ for all values of t

Table 3, Table 6, Table 9and Table 12 show Time dependent System performance measures for several values of t, λ, μ, α, p, a and b.

We infer the following

- As the value of t increases and for several values of t, λ, μ, α, p, a and b $P_{busy}(t)\to P_{busy}, P_{vacation}(t)\to P_{vacation}$ and $L_q(t)\to L_q$

Table 1: Transient probability distribution of number of consumers in the queue when the server is busy

for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 2 and b = 3.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$	$P_{71}(t)$	$P_{81}(t)$
1	0.0453	0.0167	0.0058	0.0020	0.0007	0.0002	0.0001	0.0000	0.0000
2	0.0492	0.0188	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000
3	0.0494	0.0189	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000
4	0.0494	0.0189	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000
5	0.0494	0.0189	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000
6	0.0494	0.0189	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000
7	0.0494	0.0189	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000
8	0.0494	0.0189	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000
9	0.0494	0.0189	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000
10	0.0494	0.0189	0.0069	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000

Table 2: Transient probability distribution of number of consumers in the queue when the server is in vacation for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 2 and b = 3.

t	$P_{02}(t)$	$P_{12}(t)$	$P_{22}(t)$	$P_{32}(t)$	$P_{42}(t)$	$P_{52}(t)$	$P_{62}(t)$	$P_{72}(t)$	$P_{82}(t)$
1	0.3541	0.4465	0.1007	0.0221	0.0046	0.0009	0.0002	0.0000	0.0000
2	0.3303	0.4548	0.1049	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000
3	0.3292	0.4552	0.1050	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000
4	0.3292	0.4552	0.1051	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000
5	0.3292	0.4552	0.1051	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000
6	0.3292	0.4552	0.1051	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000
7	0.3292	0.4552	0.1051	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000
8	0.3292	0.4552	0.1051	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000
9	0.3292	0.4552	0.1051	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000
10	0.3292	0.4552	0.1051	0.0242	0.0056	0.0013	0.0003	0.0001	0.0000

Table 3: System performance measures for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 2 and b = 3.

t	Pbusy(t)	Pvacation(t)	Lq(t)
1	0.0709	0.9291	0.7775
2	0.0786	0.9214	0.8139
3	0.0790	0.9210	0.8155
4	0.0790	0.9210	0.8156
5	0.0790	0.9210	0.8156
6	0.0790	0.9210	0.8156
7	0.0790	0.9210	0.8156
8	0.0790	0.9210	0.8156
9	0.0790	0.9210	0.8156
10	0.0790	0.9210	0.8156

Table 4: Transient probability distribution of number of consumers in the queue when the server is busy for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 3 and b = 3.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$	$P_{71}(t)$	$P_{81}(t)$
1	0.0207	0.0107	0.0042	0.0015	0.0005	0.0002	0.0001	0.0000	0.0000
2	0.0281	0.0159	0.0068	0.0026	0.0010	0.0003	0.0001	0.0000	0.0000
3	0.0287	0.0164	0.0071	0.0028	0.0010	0.0004	0.0001	0.0000	0.0000
4	0.0286	0.0164	0.0071	0.0028	0.0010	0.0004	0.0001	0.0000	0.0000
5	0.0286	0.0164	0.0071	0.0028	0.0010	0.0004	0.0001	0.0000	0.0000
6	0.0286	0.0164	0.0071	0.0028	0.0010	0.0004	0.0001	0.0000	0.0000
7	0.0286	0.0164	0.0071	0.0028	0.0010	0.0004	0.0001	0.0000	0.0000
8	0.0286	0.0164	0.0071	0.0028	0.0010	0.0004	0.0001	0.0000	0.0000
9	0.0286	0.0164	0.0071	0.0028	0.0010	0.0004	0.0001	0.0000	0.0000
10	0.0286	0.0164	0.0071	0.0028	0.0010	0.0004	0.0001	0.0000	0.0000

Table 5: Transient probability distribution of number of consumers in the queue when the server is in vacation for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 3 and b = 3

t	$P_{02}(t)$	$P_{12}(t)$	$P_{22}(t)$	$P_{32}(t)$	$P_{42}(t)$	$P_{52}(t)$	$P_{62}(t)$	$P_{72}(t)$	$P_{82}(t)$
1	0.2347	0.3529	0.2967	0.0624	0.0126	0.0024	0.0004	0.0001	0.0000
2	0.1888	0.3071	0.3465	0.0793	0.0181	0.0041	0.0009	0.0002	0.0000
3	0.1901	0.3003	0.3484	0.0804	0.0186	0.0043	0.0010	0.0002	0.0001
4	0.1908	0.3002	0.3480	0.0803	0.0185	0.0043	0.0010	0.0002	0.0001
5	0.1909	0.3003	0.3479	0.0803	0.0185	0.0043	0.0010	0.0002	0.0001
6	0.1909	0.3003	0.3479	0.0803	0.0185	0.0043	0.0010	0.0002	0.0001
7	0.1909	0.3003	0.3479	0.0803	0.0185	0.0043	0.0010	0.0002	0.0001
8	0.1909	0.3003	0.3479	0.0803	0.0185	0.0043	0.0010	0.0002	0.0001
9	0.1909	0.3003	0.3479	0.0803	0.0185	0.0043	0.0010	0.0002	0.0001
10	0.1909	0.3003	0.3479	0.0803	0.0185	0.0043	0.0010	0.0002	0.0001

Table 6: System performance measures for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 3 and b = 3

t	Pbusy(t)	Pvacation(t)	Lq(t)
1	0.0378	0.9622	1.2257
2	0.0549	0.9451	1.3822
3	0.0567	0.9433	1.3884
4	0.0566	0.9434	1.3872
5	0.0566	0.9434	1.3870
6	0.0566	0.9434	1.3870
7	0.0566	0.9434	1.3870
8	0.0566	0.9434	1.3870
9	0.0566	0.9434	1.3870
10	0.0566	0.9434	1.3870

Table 7: Transient probability distribution of number of consumers in the queue when the server is busy

for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 4 and b = 6.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$	$P_{71}(t)$	$P_{81}(t)$
1	0.0118	0.0036	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
2	0.0240	0.0081	0.0027	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
3	0.0257	0.0088	0.0030	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
4	0.0252	0.0086	0.0029	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
5	0.0251	0.0086	0.0029	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
6	0.0251	0.0086	0.0029	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
7	0.0251	0.0086	0.0029	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
8	0.0251	0.0086	0.0029	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
9	0.0251	0.0086	0.0029	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
10	0.0251	0.0086	0.0029	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000

Table 8: Transient probability distribution of number of consumers in the queue when the server is in vacation for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 4 and b = 6

t	$P_{02}(t)$	$P_{12}(t)$	$P_{22}(t)$	$P_{32}(t)$	$P_{42}(t)$	$P_{52}(t)$	$P_{62}(t)$	$P_{72}(t)$	$P_{82}(t)$
1	0.1978	0.3117	0.2707	0.1642	0.0316	0.0058	0.0010	0.0002	0.0000
2	0.1499	0.2239	0.2641	0.2530	0.0569	0.0126	0.0028	0.0006	0.0001
3	0.1654	0.2196	0.2439	0.2552	0.0591	0.0137	0.0032	0.0007	0.0002
4	0.1681	0.2244	0.2430	0.2508	0.0580	0.0134	0.0031	0.0007	0.0002
5	0.1674	0.2249	0.2441	0.2505	0.0578	0.0133	0.0031	0.0007	0.0002
6	0.1672	0.2246	0.2442	0.2507	0.0578	0.0133	0.0031	0.0007	0.0002
7	0.1672	0.2246	0.2441	0.2507	0.0579	0.0134	0.0031	0.0007	0.0002
8	0.1672	0.2246	0.2441	0.2507	0.0579	0.0134	0.0031	0.0007	0.0002
9	0.1672	0.2246	0.2441	0.2507	0.0579	0.0134	0.0031	0.0007	0.0002
10	0.1672	0.2246	0.2441	0.2507	0.0579	0.0134	0.0031	0.0007	0.0002

Table 9: System performance measures for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a=4 and b=6

t	Pbusy(t)	Pvacation(t)	Lq(t)
1	0.0169	0.9831	1.5163
2	0.0360	0.9640	1.8419
3	0.0390	0.9610	1.8237
4	0.0383	0.9617	1.8069
5	0.0381	0.9619	1.8072
6	0.0381	0.9619	1.8081
7	0.0381	0.9619	1.8082
8	0.0381	0.9619	1.8081
9	0.0381	0.9619	1.8081
10	0.0381	0.9619	1.8081

Table 10: Transient probability distribution of number of consumers in the queue when the server is busy forvarious values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 5 and b = 6.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$	$P_{71}(t)$	$P_{81}(t)$
1	0.0047	0.0015	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0157	0.0056	0.0019	0.0006	0.0002	0.0001	0.0000	0.0000	0.0000
3	0.0203	0.0076	0.0027	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
4	0.0199	0.0075	0.0027	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
5	0.0193	0.0073	0.0026	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
6	0.0193	0.0073	0.0026	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
7	0.0194	0.0073	0.0026	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
8	0.0194	0.0073	0.0026	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
9	0.0194	0.0073	0.0026	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
10	0.0194	0.0073	0.0026	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000

Table 11: Transient probability distribution of number of consumers in the queue when the server is in vacation for various values of t, λ = 5, μ =10, α = 5, p = 0.3,a = 5 and b = 6.

t	$P_{02}(t)$	$P_{12}(t)$	$P_{22}(t)$	$P_{32}(t)$	$P_{42}(t)$	$P_{52}(t)$	$P_{62}(t)$	$P_{72}(t)$	$P_{82}(t)$
1	0.1764	0.2976	0.2643	0.1618	0.0769	0.0136	0.0023	0.0004	0.0001
2	0.0999	0.1742	0.2295	0.2333	0.1879	0.0403	0.0085	0.0018	0.0004
3	0.1220	0.1634	0.1901	0.2124	0.2158	0.0496	0.0113	0.0026	0.0006
4	0.1320	0.1768	0.1905	0.1992	0.2072	0.0481	0.0112	0.0026	0.0006
5	0.1305	0.1796	0.1956	0.2000	0.2027	0.0468	0.0108	0.0025	0.0006
6	0.1291	0.1783	0.1961	0.2018	0.2032	0.0469	0.0108	0.0025	0.0006
7	0.1291	0.1778	0.1956	0.2019	0.2038	0.0470	0.0108	0.0025	0.0006
8	0.1293	0.1779	0.1954	0.2017	0.2038	0.0470	0.0109	0.0025	0.0006
9	0.1293	0.1780	0.1955	0.2016	0.2038	0.0470	0.0109	0.0025	0.0006
10	0.1293	0.1780	0.1955	0.2016	0.2038	0.0470	0.0109	0.0025	0.0006

Table 12: System performance measures for various values of t, λ = 5, μ =10, α = 5, p = 0.3, a = 5 and b = 6.

t	Pbusy(t)	Pvacation(t)	Lq(t)
1	0.0068	0.9932	1.7071
2	0.0241	0.9759	2.3660
3	0.0321	0.9679	2.4021
4	0.0317	0.9683	2.3346
5	0.0307	0.9693	2.3222
6	0.0306	0.9694	2.3290
7	0.0307	0.9693	2.3313
8	0.0307	0.9693	2.3308
9	0.0307	0.9693	2.3304
10	0.0307	0.9693	2.3304

VII. CONCLUSION

A new computational approach was used to evaluate the Transient behaviour of Bulk service queueing system with multiple vacation and discouraged customer model using infinite generator matrix and Hamilton theorem. Numerical studies have been analysed in elaborate manner. In this model we have provided transient probability distribution of number of consumers in the queue at time t and also time dependent system measures.

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