BUSY TIME FOR GROUP ARRIVAL AND GROUP SERVICE MODELS

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Abstract

The motive of this paper is to investigate and analysisof the distributions of the number of customers waiting in line using Erlangian techniques,the generating functions and the Integral transforms. Here the arrivals of unit sizes occur in groups according to a homogeneous Poisson process. The service time of customers is a random variable and is independent of the number of customers in the batch.

1. INTRODUCTION:

In queuing theory, a bulk queue is a general queuing model where jobs arrive in and are served in groups of random size. The network of such queues is known to have a product from stationary distribution under certain conditions. They behave like Brownian Reflection under high traffic conditions.

The busy period is the time interval that begins with an arrival of an individual who activates the idle server and ends when for the first time thereafter, the server becomes idle again. The probability distribution of the number served in the busy period and the duration of the busy period are of significant interest.

There are various methods to study mathematically some characteristics of the congestion theory. Neutus (1966) considered imbedded Markov and Semi analysis of a bulk queue. Cohen (1963) considered an application of derived Markov chain in queueing theory. Jaiswal(1960) pointed out that from a practical point of view it is preferable to find solution to a queueing problem by Erlangian procedure or its modification. Choudhary and Templeton (1972) analyzed the theory of bulk arrival and bulk service for some aspects of a single server queue.

A queue may be comprised under the title of a dual bulk queue if

(i) The service is in a group following a probability distribution.
(ii) The arrivalsare in groups following given probability distribution.

Hiller (1959) first initiated the analysis of a dual bulk queue who used the imbedded Markov chain technique to discuss some properties related to the dual bulk queue. Kailson (1962) used the phase space method. Bhat (1964) used the results from Fluctuation theory to analyze the equilibrium distribution of the queue length and busy period.

Rao (1968) employed the phase method or a modified Erlangian procedure to consider queue length for a transportation type dual bulk queue with an arbitrary arrival time distribution and exponential service time distribution.

Here we have presented some aspects of a single server dual bulk queue by the modified Erlangian procedure in a general manner.

2. ASSUMPTIONS AND NOTATIONS:

2.1 Kendall's notation:

In Kendall's notation for single queuing nodes, the random variable denoting bulk arrivals or services is denoted with a superscript, for example

$M^X|M^Y|1$ denotes an M|M|1 queue where the arrivals are in batches determined by the random variable X and service in bulk determined by the random variable Y.

2.2 Bulk Service:

Customers arrive at random instants according to a Poisson process and form a single queue, from the front of which batches of customers are served at a rate with independent distribution. The equilibrium distribution, mean, and variance of queue length are known for this mode. The optimal maximum size of the batch, subject to operating cost constraints, can be modeled as a Markov decision process.

2.3 Bulk Arrival:

Optimal service provision procedure to minimize long-run expected cost.

2.4 Arrival Process:

The arrivals of units occur in groups according to a homogenous Poisson process with arrival rate **λ.** The group size X is a random variable and,

$$a_r = P\{ X^m . r\} , \qquad r=1,2,3$$

It is also assumed that the distribution for the number of customers who arrive at each arrival period of notable events has finite mean $\bar{a}$ and C_a^2 with generating function

$$A(X) = a_m X^m \sum_{m=1}^{\infty}$$

2.5 **Queue Discipline :**

The queue discipline is first come first out. This means customers first entering into the queue will get out of the queue into the service channel for service. The order in which the members of a particular group present themselves to the server is a matter of insignificance.

2.6 Service Mechanism :

The service time of customers is a random variable having a probability distribution function

$$\mu\,(\mu t)^{r-1j} \quad C_r \;\_\_\_\_\_\_\_\_ \quad e^{-\mu t}$$
$$(r-1)! \quad \sum_{r=1}$$

and is independent of the customers in the batch. C_r, r=1,2,3….. being the probability that a batch of S or the entire queue whichever is lesser is taken in the rth phase each having exponential service time with a parameter μ. Itis further assumed that among the customers who arrive where the server is idle, S are taken immediately for service if their number of arrivals is greater than S, otherwise, all are taken for service.

2.7 Traffic Intensity :

Let the arrival and service assumptions be expressed in terms of the traffic intensity

$$\boldsymbol{\lambda}\ \bar{a}\Sigma r C_r \,\_\_\_\_\_\_\_\_\_\_$$
$$\mu_s$$

which will be assumed to be less than unity in the case of steady-state.

3DERIVATIONS OF THE EQUATIONS :

Here we have,

$P_{n,r}(t)$ = Probability that n customers are waiting in the queue andthe service is in the rth phase at time t.

$P_o(t)$ = Probability that no customers is waiting in the queue and the server is free i.e. the system is empty at time t.

The difference differential equations for the queuing procedure are as follows:

$P'_o(t) = -\lambda P_o(t) + \mu P_{o,1}(t)$

$P'_{o,r}(t) = -(\lambda+\mu) P_{o,r}(t) + \mu P_{o,r+1}(t) + \mu C_r \sum_{i=1}^{S} P_{i,1}(t) + \lambda C_r \sum_{i=1}^{S} a_i P_0(t)$

$P'_{o,j}(t) = -(\lambda+\mu) P_{o,j}(t) + \mu C_j \sum_{i=1}^{S} P_{i,1}(t) + \lambda C_j \sum_{i=1}^{S} a_i P_0(t)$

$P'_{n,r}(t) = -(\lambda+\mu) P_{n,r}(t) + \lambda \sum_{m=1}^{n} a_m P_{n-m,r}(t) + \lambda P_{m,r+1}(t) + \mu C_r P_{n+s,1}(t) + \lambda C_r$

$a_{n+s} P_0(t)$ $(n>0, \leq r<j)$

$P'_{n,j}(t) = -(\lambda+\mu) P_{n,j}(t) + \lambda C_j \sum_{m=1}^{n} a_m P_{n-m,j}(t) + \mu C_j P_{n+s,1}(t) + + \lambda C_j a_{n+s} P_0(t)$

$,n>0$, where $P'_{n,r}(t) = \frac{d\{P_{n+s}(t)\}}{dt}$ etc.

3. Solution of Equations:

We denote the Laplace transform of $P_{n,r}(t)$ by $P'_{n,r}(\alpha)$. Then

$P'_{n,r}(\alpha) = \int_0^{\infty} exp.(-\alpha t) P_{n,r}(t)\ dt$

Also, let us assume that the process starts with the system being empty, i.e.

$P_0(0)=1$

We further introduce the following generating function

$A(\gamma) = \sum_{r=1}^{\infty} a_r \gamma^r$

$P_n(\gamma,\alpha) = \sum_{r=1}^{j} \bar{P'}_{n,r}(\alpha)\gamma^r$

$H(\gamma,X,\alpha)=\sum_{n=0}^{\infty} P_n(\gamma,\alpha)X^n \quad =\sum_{n=0}^{\infty} \quad \sum_{r=0}^{j} \bar{P}_{n,r}(\alpha)X^n\gamma^r$

Taking the Laplace transform of the basic equation multiplying by the appropriate powers of X and γ and summing out the values of n and , the following equation is obtained:

$$\left[\alpha+\lambda+\mu-A(X)-\frac{\mu}{y}\right] H(\gamma,X,\alpha)+\mu \left[1-\frac{C(y)}{X^S}\right]\sum_{n=0}^{\infty}\bar{P}_{n,1}(\alpha)X^n -- \left[\frac{C(y)}{X^S}\right]\left[\sum_{i=1}^{S-1}\left\{\mu\bar{P}_{i,1}(\alpha)+\lambda a_1\bar{P}_0(\alpha)\right\}\right](X^s-X')+1-(\alpha+\lambda-\lambda A(x)\bar{P}_{0(\alpha)})=0$$

......(1)

Where $\sum_{r=1}^{j} \sum_{n=1}^{\infty} \sum_{m=1}^{n} a_m P_{n-m,r}(\alpha)X^{n-m}\gamma^r = A(X)\, H(\gamma,X,\alpha)$

And $C(\gamma)=\sum_{r=1}^{j} C_r\gamma^r$

The above equation is valid for |X|<1 and |Y|<1, it is also valid for

$\gamma= \mu/ \alpha+\lambda+\mu-\lambda\Lambda(X)$, because $R_e(\alpha)>0$.

Thus we get from $\sum_{n=0}^{\infty} P_{n,1}(\alpha)X^n =$

Type equation here.

$$\frac{\sum_{i=1}^{S-1}\{\mu\bar{P}_{i,1}(\alpha)+\lambda a_1\bar{P}_0(\alpha)\}(Xs-X')+1-(\alpha+\lambda-\lambda A(x)\bar{P}_0(\alpha))}{\mu\left[\frac{X^S}{C(y)}-1\right]}$$

........(2)

where γ is given by $\gamma= \mu/ \alpha+\lambda+\mu-\lambda\Lambda(X)$

if we denote $C(\mu/\ \alpha+\lambda+\mu-\lambda\Lambda(X))$ by B(x)

and $\sum_{n=0}^{\infty}\bar{P}_{n,1}(\alpha)X^n$ by e_j (X,α), then by putting (2) in (1) we get after substituting γ=1, $H(1,X,\alpha)=\sum_{n=0}^{\infty}P_{n,1}(\alpha)X^n$

…….(3)

$H(1,X,\alpha)=\frac{\mu(\frac{1}{B(X)}-1)}{\alpha+\lambda-\lambda_j\Lambda(X)}G(X,\alpha)$ where $P_n(1,\alpha)\sum_{r=1}^{j}\bar{P}_{n,r}(\alpha)$

is the Laplace transform of the probability that n units are waiting in the queue at time t.H(1,X,α) is the generalization of the result obtained by Jaiswal(1960)

5. Conclusion :

The result may be applied to batch arrivals. A general class of queue having real-time arrivals with arbitrary independent service distributions has also been considered. There are many queueing activities in which arrivals and service can be in a group i.e. in batches or bulk. Several people may go to a restaurant together and obtain service as a batch. Several long-distance telephone calls may present themselves simultaneously before an operator. The result can be applied in these types of queueing problems