# COMPARATIVE STUDY OF SEVERAL METHODS TO OBTAIN AN INITIAL BASIC FEASIBLE SOLUTION OF TRANSPORTATION PROBLEM

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**ABSTRACT:**
Transportation problem is one of the most important areas of operation research and it is useful for decision making of transportation of goods. Transportation problem is concerned with determining an optimal strategy for transporting goods from a number of origins or sources to various destination in a such way that the total transportation cost is minimized. That is to determine how to transport goods in optimal way. In this paper we compare several methods for determine the initial basic feasible solution (IBFS) of transportation problem .

KEYWORD: Transportation Problem, IBFS, Supply, Demand, Goods

**INTRODUCTION**:

Transportation problem is a type of linear programming problem, which is associated with day-to-day activities in daily life and deals with logistics. Transportation problems deal with the transportation of a single product manufactured at different factories plants (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy supply and demand condition and minimum transportation cost possible.To achieve this objective, we must know the quantity of available supplies and the quantities demanded. The main role of this transportation table is to minimize total transportation cost of transporting goods from origins to supply. Following are some well known techniques which are used for obtaining minimum transportation cost .

a) North –West Corner Method
b) Least Cost Method
c) Row Minima Method
d) Column Minima Method
e) Vogel's Approximation Method

f) Average Opportunity Cost Method

These methods gives initial Basic feasible Solution to transportation problem and its is optimality is checked by MODI method. Now there are two types transportation problem one is balanced Transportation Problem and another once is unbalanced Transportation Problem. If total supply is equal to total demand then it is balanced transportation problem.Otherwise it is unbalanced transportation problem. In this paper we have compared initial Basic feasible Solutions obtained by these methods by taking numerical example and given the opinion that which method gives better solution.

**MATHEMATICAL FORMULATION:**

The transportation problem is shown as a linear transportation model as below.

Minimize ,

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} X_{ij}$$

Where, $\sum_{j=1}^{m} X_{ij} = a_i$ , i=1,2,……,n (supply) and $\sum_{i=1}^{n} X_{ij} = b_j$ , j=1,2,……,m (demand)

And $X_{ij} \geq 0$, for all i and j.

Where,

$X_{ij}$=The quantity to be shipped from ith origin to jth destination.

$C_{ij}$=per piece(unit) cost in shipping from ith origin to jth destination.

$a_i$= The amount available at ith origin.

$b_j$=The demand available at jth destination

**Algorithm of Methods of Solving Transportation Problem:**

**a) North –West Corner Method**

**Steps in North –West Corner Method :**

1. Select the upper left-hand corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand, i.e., min(s1, d1).
2. Adjust the supply and demand numbers in the respective rows and columns.

3. If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.
4. If the supply for the first row is exhausted, then move down to the first cell in the second row.
5. If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column.
6. Continue the process until all supply and demand values are exhausted.

**b) Least Cost Method**

**Steps in Least Cost Method :-**

**Step 1**: Find the cell with the least(minimum) cost in the transportation table.

**Step 2**: Allocate the maximum feasible quantity to this cell.

**Step:3:** Eliminate the row or column where an allocation is made.

**Step:4:** Repeat the above steps for the reduced transportation table until all the allocations are made.

**c) Row Minima Method**

Step 1: In this method, we allocate as much as possible in the lowest cost cell of the first row, i.e. allocate min($s_i$, **dj**).

Step 2: a) Subtract this min value from supply $s_i$ and demand **dj**.

b) If the supply $s_i$ is 0, then cross (strike) that row and if the demand **dj** is 0 then cross (strike) that column.

c) If min unit cost cell is not unique, then select the cell where maximum

Step-3: Repeat this process for all uncrossed (unstriked) rows and columns until all supply and demand values are 0.

**d) Column Minima Method**

Step 1: In this method, we allocate as much as possible in the lowest cost cell of the first Column, i.e. allocate min ($s_i$, **dj**).

Step 2: a) Subtract this min value from supply $s_i$ and demand **dj**.

b) If the supply $s_i$ is 0, then cross (strike) that row and If the demand **dj** is 0 then Cross (strike) that column.

c) If min unit cost cell is not unique, then select the cell where maximum allocation Can be possible.

Step-3: Repeat this process for all uncrossed (unstriked) rows and columns until all supply and demand values are 0.

**e) Vogel's Approximation Method**

**Vogel's Approximation Method (VAM) Steps :-**

Step-1: Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.

Step-2: Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in each column penalty.

Step-3: Select the row or column with the maximum penalty and find cell that has least cost in selected row or column. Allocate as much as possible in this cell. If there is a tie in the values of penalties then select the cell where maximum allocation can be possible.

Step-4: Adjust the supply and demand and cross out (strike out) the satisfied row or column.

Step-5: Repeat this steps until all supply and demand values are 0.

**f) Average Opportunity Cost Method**

**ALGOTITHM OF AOCM**

STEP 1: Subtract smallest cost from every element of every row of transportation table and put it on right top of that element.

STEP 2: Subtract the smallest cost from every element of every column of transportation table and put it on right bottom of that element.

STEP 3: Create a new matrix whose elements are average value of right top and right bottom of elements of step 1 and step 2.

STEP 4: Find Row and Column penalties by taking difference between smallest & next smallest value in row and column.

STEP 5: Identify the largest penalty and allocate the maximum possible quantity to that Cell having minimum value of element in corresponding row or column.

## IV.NUMERICAL EXAMPLES

4.1 Solve the following Transportation Problem.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 19 | 30 | 50 | 10 | 7 |
| $S_2$ | 70 | 30 | 40 | 60 | 9 |
| $S_3$ | 40 | 8 | 70 | 20 | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

**Solution:**

**a) North –West Corner Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 19 (5) | 30 (2) | 50 | 10 | 7 |
| $S_2$ | 70 | 30 (6) | 40 (3) | 60 | 9 |
| $S_3$ | 40 | 8 | 70 (4) | 20 (14) | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

The Total Transportation Cost =19*5 +30*2+30*6+40*3+70*4+20*14=1015

**b) Least Cost Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 19 | 30 | 50 | 10 (7) | 7 |
| $S_2$ | 70 (2) | 30 | 40 (7) | 60 | 9 |
| $S_3$ | 40 (3) | 8 (8) | 70 | 20 (7) | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

The Total Transportation Cost =10*7 +70*2+40*7+40*3+8*8+20*7=814

**c) Row Minima Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 19 | 30 | 50 | 10 (7) | 7 |
| $S_2$ | 70 | 30 (8) | 40 (1) | 60 | 9 |
| $S_3$ | 40 (5) | 8 | 70 (6) | 20 (7) | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

The Total Transportation Cost =10*7 +30*8+40*1+40*5+70*6+20*7=1110

**d) Column Minima Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 19 (5) | 30 | 50 | 10 (2) | 7 |
| $S_2$ | 70 | 30 | 40 (7) | 60 (2) | 9 |
| $S_3$ | 40 | 8 (8) | 70 | 20 (10) | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

The Total Transportation Cost =19*5 +10*2+40*7+60*2+8*8+20*10=779

**e) Vogel's Approximation Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $S_1$ | 19 (5) | 30 | 50 | 10 (2) | 7,2,0 | 9 | 9 | 40 | 40 | - | - |
| $S_2$ | 70 | 30 | 40 (7) | 60 (2) | 9,7,0 | 10 | 20 | 20 | 20 | 20 | 40 |
| $S_3$ | 40 | 8 (8) | 70 | 20 (10) | 18,10,0 | 12 | 20 | 50 | - | - | - |
| Demand | 5,0 | 8,0 | 7,0 | 14,4,2,0 | 34 | | | | | | |
| $P_1$ | 21 | 22 | 10 | 10 | | | | | | | |
| $P_2$ | 21 | - | 10 | 10 | | | | | | | |
| $P_3$ | - | - | 10 | 10 | | | | | | | |
| $P_4$ | - | - | 10 | 50 | | | | | | | |
| $P_5$ | - | - | 40 | 60 | | | | | | | |
| $P_6$ | - | - | 40 | - | | | | | | | |

The Total Transportation Cost =19*5 +10*2+40*7+60*2+8*8+20*10=779

**f) Average Opportunity Cost Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 19 | 30 | 50 | 10 | 7 |
| $S_2$ | 70 | 30 | 40 | 60 | 9 |
| $S_3$ | 40 | 8 | 70 | 20 | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

Substracting Smallest cost of every Row from each element of that Row

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 9 | 20 | 40 | 0 | 7 |
| $S_2$ | 40 | 0 | 10 | 30 | 9 |
| $S_3$ | 32 | 0 | 62 | 12 | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

Substracting Smallest cost of every Column from each element of that Column

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 0 | 22 | 10 | 0 | 7 |
| $S_2$ | 51 | 22 | 0 | 50 | 9 |
| $S_3$ | 21 | 0 | 30 | 10 | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

Taking average of both the table and calculating penalties

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $S_1$ | 4.5 (5) | 21 | 25 | 0 (2) | 7,2,0 | 4.5 | 21 | - | - | - |
| $S_2$ | 45.5 | 11 (2) | 5 (7) | 40 | 9,2,0 | 6 | 6 | 6 | 29 | 11 |
| $S_3$ | 26.5 | 0 (6) | 46 | 11 (12) | 18,6,0 | 11 | 11 | 11 | 11 | 0 |
| Demand | 5,0 | 8,2,0 | 7,0 | 14,12,0 | 34 | | | | | |
| $P_1$ | 22 | 11 | 20 | 11 | | | | | | |
| $P_2$ | - | 11 | 20 | 11 | | | | | | |
| $P_3$ | - | 11 | 41 | 29 | | | | | | |
| $P_4$ | - | 11 | - | 29 | | | | | | |
| $P_5$ | - | 11 | - | - | | | | | | |

Now putting allocation value in original matrix

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 19 (5) | 30 | 50 | 10 (2) | 7 |
| $S_2$ | 70 | 30 (2) | 40 (7) | 60 | 9 |
| $S_3$ | 40 | 8 (6) | 70 | 20 (12) | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

The Total Transportation Cost =19*5 +10*2+30*2+40*7+8*6+20*12=743

4.2 Solve the following Transportation Problem.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 | 10 | 4 | 5 | 10 |
| $S_2$ | 6 | 8 | 7 | 2 | 25 |
| $S_3$ | 4 | 2 | 5 | 7 | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

**Solution:**

**a) North –West Corner Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 (10) | 10 | 4 | 5 | 10 |
| $S_2$ | 6 (15) | 8 (10) | 7 | 2 | 25 |
| $S_3$ | 4 | 2 | 5 (15) | 7 (5) | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

The Total Transportation Cost =5*10+6*15+8*10+5*15+7*5=330

**b)Least Cost Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 | 10 | 4 (10) | 5 | 10 |
| $S_2$ | 6 (15) | 8 | 7 (5) | 2 (5) | 25 |
| $S_3$ | 4 (10) | 2 (10) | 5 | 7 | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

The Total Transportation Cost =4*10+6*15+7*5+2*5+4*10+2*10=235

**c) Row Minima Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 | 10 | 4 (10) | 5 | 10 |
| $S_2$ | 6 (20) | 8 | 7 | 2 (5) | 25 |
| $S_3$ | 4 (5) | 2 (10) | 5 (5) | 7 | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

The Total Transportation Cost=4*10+6*20+2*5+4*5+2*10+5*5=235

### d) Column Minima Method

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 (5) | 10 | 4 (5) | 5 | 10 |
| $S_2$ | 6 | 8 (10) | 7 (10) | 2 (5) | 25 |
| $S_3$ | 4 (20) | 2 | 5 | 7 | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

The Total Transportation Cost= 5*5+4*20+8*10+4*5+7*10+2*5=285

### e) Vogel's Approximation Method

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $S_1$ | 5 | 10 | 4 (10) | 5 | 10,0 | 1 | 1 | 1 | 1 | - |
| $S_2$ | 6 (20) | 8 | 7 | 2 (5) | 25,20,0 | 4 | 4 | 1 | - | - |
| $S_3$ | 4 (5) | 2 (10) | 5 (5) | 7 | 20,10,5,0 | 2 | 1 | 1 | 1 | 1 |
| Demand | 25,5,0 | 10,0 | 15,5,0 | 5,0 | | | | | | |
| $P_1$ | 1 | 6 | 1 | 3 | | | | | | |
| $P_2$ | 1 | - | 1 | 3 | | | | | | |
| $P_3$ | 1 | - | 1 | - | | | | | | |
| $P_4$ | 1 | - | 1 | - | | | | | | |
| $P_5$ | 4 | - | 5 | - | | | | | | |

The Total Transportation Cost =4*10+6*20+2*5+4*5+2*10+5*5=235

**f) Average Opportunity Cost Method**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 | 10 | 4 | 5 | 10 |
| $S_2$ | 6 | 8 | 7 | 2 | 25 |
| $S_3$ | 4 | 2 | 5 | 7 | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

Subtracting Smallest cost of every Row from each element of that Row

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 1 | 6 | 0 | 1 | 10 |
| $S_2$ | 4 | 6 | 5 | 0 | 25 |
| $S_3$ | 2 | 0 | 3 | 5 | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

Substracting Smallest cost of every Column from each element of that Column

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 1 | 8 | 0 | 3 | 10 |
| $S_2$ | 2 | 6 | 3 | 0 | 25 |
| $S_3$ | 0 | 0 | 1 | 5 | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

Taking average of both the table and calculating penalties

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|---|---|---|
| $S_1$ | 1 | 7 | 0 (10) | 2 | 10,0 | 1 | 1 | 1 | 1 |
| $S_2$ | 3 (15) | 6 | 4 (5) | 0 (5) | 25,20,15,0 | 4 | 4 | 1 | - |
| $S_3$ | 1 (10) | 0 (10) | 2 | 5 | 20,10,0 | 2 | 1 | 1 | 1 |
| Demand | 25,15,0 | 10,0 | 15,5,0 | 5,0 | | | | | |
| $P_1$ | 1 | 6 | 1 | 3 | | | | | |
| $P_2$ | 1 | - | 1 | 3 | | | | | |
| $P_3$ | 1 | - | 1 | - | | | | | |
| $P_4$ | 1 | - | 1 | - | | | | | |
| $P_5$ | 4 | - | 5 | - | | | | | |

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 | 10 | 4 (10) | 5 | 10 |
| $S_2$ | 6 (15) | 8 | 7 (5) | 2 (5) | 25 |
| $S_3$ | 4 (10) | 2 (10) | 5 | 7 | 20 |
| Demand | 25 | 10 | 15 | 5 | 55 |

The Total Transportation Cost =4*10+6*15+7*5+2*5+4*10+2*10=235

**V) Comparison Between results of all methods**

| Sr.no. | Method | Example 1 | Example 2 |
|---|---|---|---|
| 1 | NWCM | 1015 | 330 |
| 2 | LCM | 814 | 235 |
| 3 | RMM | 1110 | 235 |
| 4 | CMM | 779 | 285 |
| 5 | VAM | 779 | 235 |
| 6 | AOCM | 743 | 235 |

**Conclusion:**

In this paper we compared several techniques used for determine the Initial Basic Feasible Solution(IBFS) of transportation problem with some numeric examples and we compare the obtained solution through these different methods. From these data we arrive at the conclusion that, from all the existing method the North West Corner Method is very simple to use but it gives worst solution compare to other. The solution obtained by Row Minima Method is near to Optimum solution .Where as the solution obtains by Least Cost Method is very near to optimum. The Vogel's Approximation Method take long calculation to get solution but it give very near solution to optimum. Also the Average Opportunity Cost Method is long but it gives optimum and better solution as compare to all other techniques.

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