DETOUR M DISTANCE OF CARTESIAN PRODUCT OF SOME SPECIAL GRAPHS

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Abstract:

In this paper we found detour M -distance of cartesian product of some special graphs, like cycle graph, complete graph, ladder graph and star graphs.

Keywords:Distance, Detour distance, Detour M - distance

I. INTRODUCTION

Let G be a nontrivial connected graph, If u,v are vertices of a connected graph G, then the **detour M -distance** is denoted by $D^M(u,v)$ and is defined as

$$D^M(u,v) = D(u,v) + \sum_{W \in D(u,v)} \deg(W) + \sum_{W \in D(u,v)} |W|$$

The **Cartesian product** of two sets A and B denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a,b) / a \in A \& b \in B\}$$

1.1 Theorem

Let C_n be any cycle graph. The detour M -distance of cartesian product of the cycle graph is $6n^2 - 1$.

Proof:

Let us prove the theorem by induction on the number of vertices in $C_n \times C_n$.

Let $n = 4$, then $C_4 \times C_4$ is the Cartesian product of cycle graph C_4 with 16 vertices. The Cartesian product of the graph C_4 will be of the form.

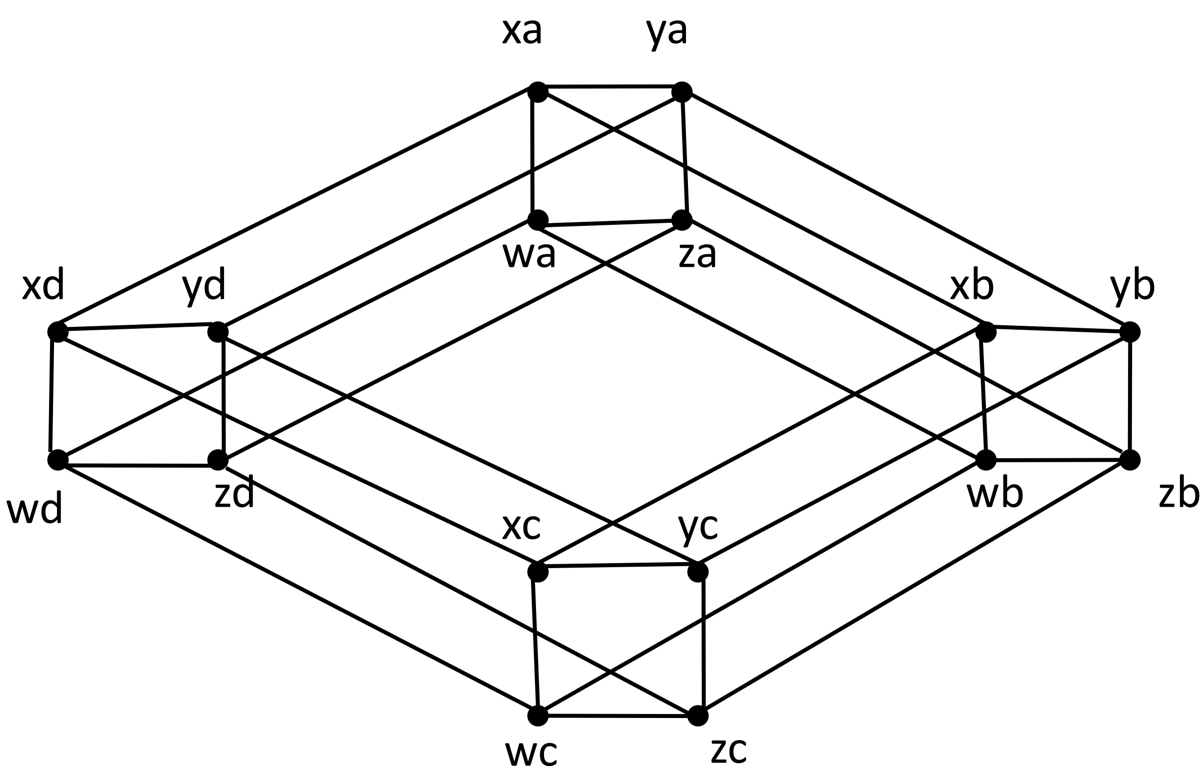

Figure 1.1

The above graph is a 4- regular graph. The detour distance between any two vertices of the graph is 15.Every vertex of the above graph has degree 4. All the vertices lie in the detour path.

By the definition of the detour M distance of the Cartesian product of the graph C_4 is

$$D^M(C_n \times C_n) = D(u,v) + \sum_{W \in D(u,v)} \deg(W) + \sum_{W \in D(u,v)} |W|$$

$$D^M(C_4 \times C_4) = 95$$

ie) $D^M(C_4 \times C_4) = 6(4)^2 - 1 = 95$

Hence the detour M distance of $C_4 \times C_4$ is 95.

Let $n = 5$, then $(C_5 \times C_5)$ is the Cartesian product of cycle graph C_5 with 25 vertices.

The Cartesian product of C_5 is of the form.

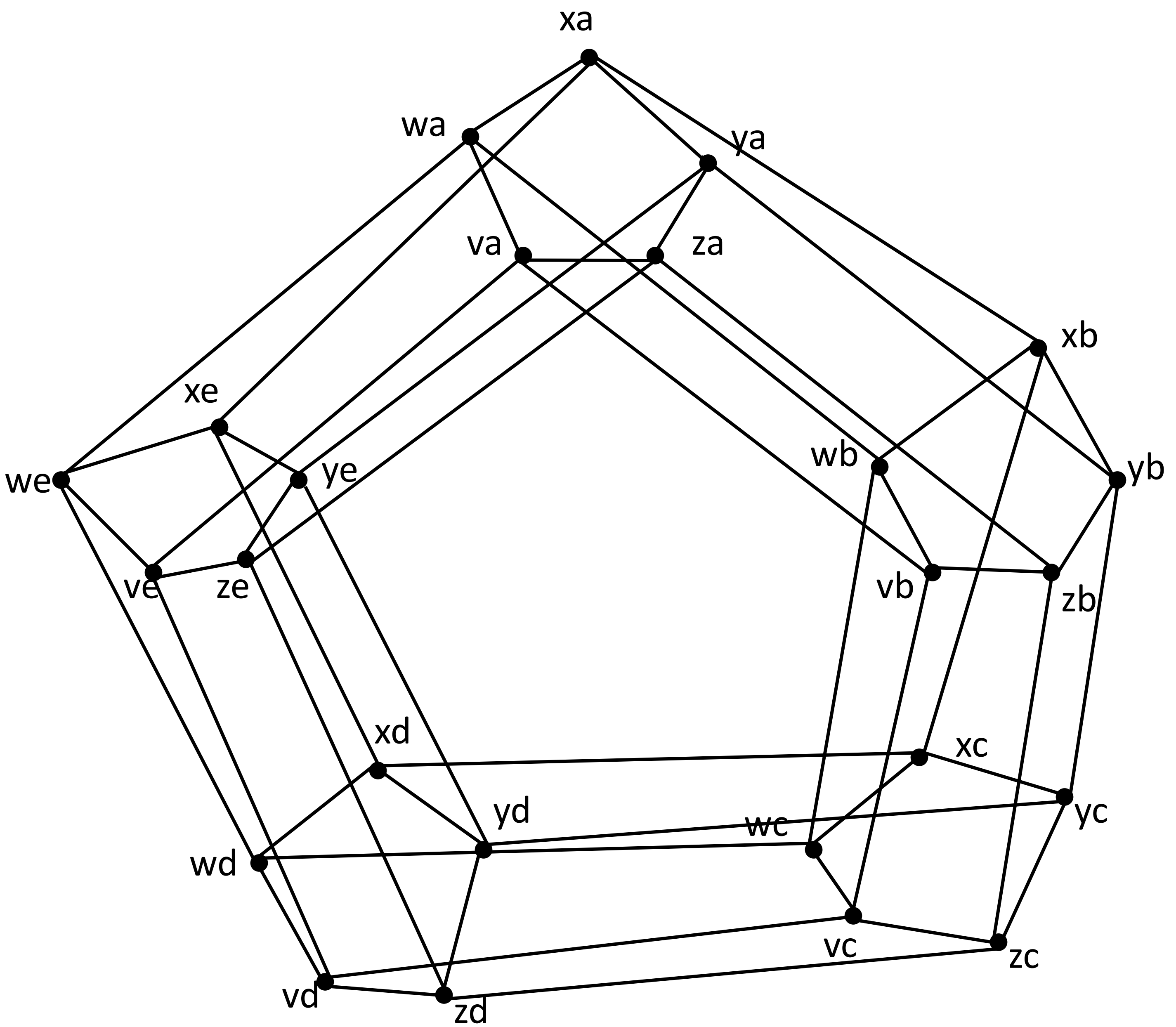

Figure 1.2

The Cartesian product of the cycle graph is also 4 regular graph. The detour distance between any two vertices of the graph is 24. All the vertices lie in the detour path.

Hence the detour M -distance of $\left(C_5 \times C_5\right)$ is 149.

Let us assume that the theorem is true for all Cartesian product of the graph C_{n-1} with $(n-1)^2$ vertices.

Now we prove the theorem for Cartesian product of the graph C_n with n^2 vertices.

Let C_n is the cycle graph. $C_n \times C_n$ be the Cartesian product of the cycle graph C_n. It has n^2 vertices. This graph is also 4-rergular graph. The detour distance between any two vertices of $C_n \times C_n$ is $n^2 - 1$.

Hence the detour M -distance between any two vertex of $C_n \times C_n$ graph is $6n^2 - 1$.

Hence the result.

1.2 Theorem

Let K_n be a complete graph. The detour M -distance of Cartesian product of complete graphs is $n^3 + 3n^2 - 1$.

Proof:

Let us prove the theorem by induction on the number of vertices in $K_n \times K_n$.

Let $n = 4$, then $K_4 \times K_4$ is the Cartesian product of complete graph K_4 with 16 vertices. The Cartesian product graph of K_4 graph will be of the form.

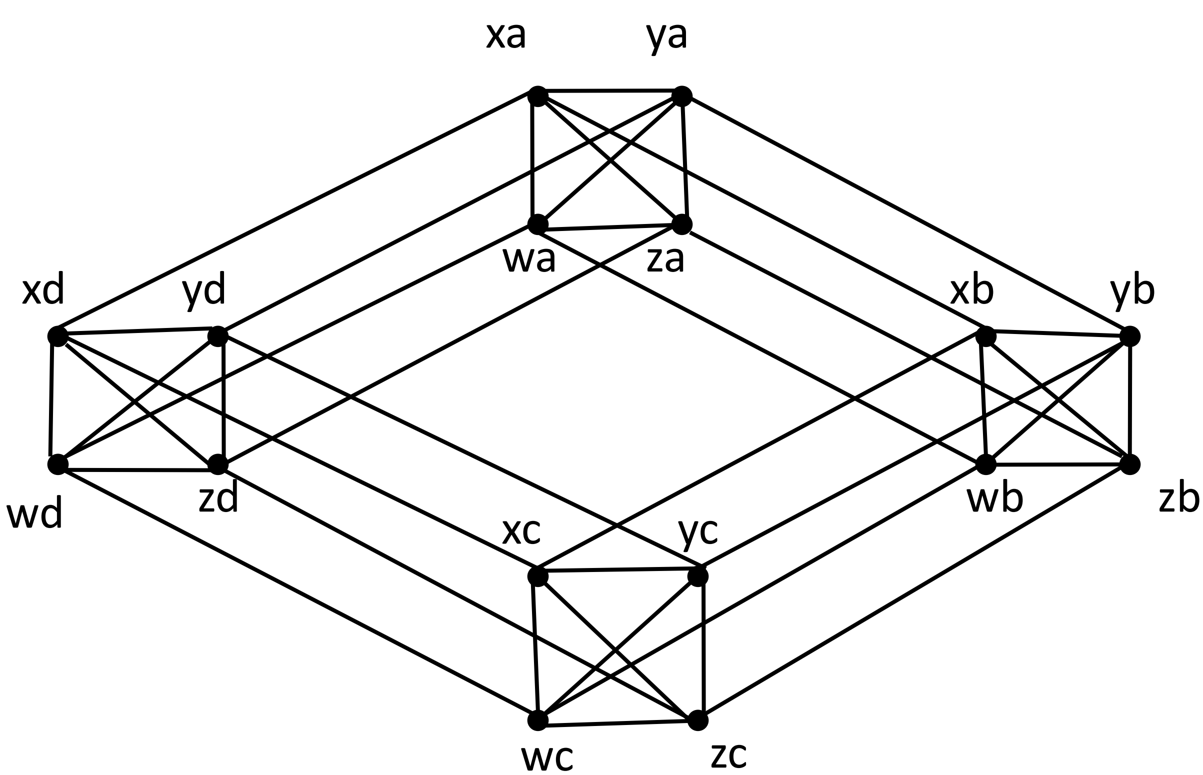

Figure 1.3

The above graph is a connected graph. Each vertex has 5 degree. The detour distance between any two vertices of the graph is 15. All the vertices lie in the detour path of the graph.

Therefore by the definition of detour M -distance,

$$D^{M}(K_n \times K_n) = D(u,v) + \sum_{W \in D(u,v)} \deg(W) + \sum_{W \in D(u,v)} |W|$$

$$D^{M}(K_4 \times K_4) = 111$$

ie) $D^{M}(K_4 \times K_4) = (4)^3 + 3(4)^2 - 1 = 111$

Hence the detour M -distance of $K_4 \times K_4$ is 111.

Let $n = 5$, then $K_5 \times K_5$ the Cartesian product of complete graph K_5 with 25 vertices. The Cartesian product graph of complete graph K_5 is of the form,

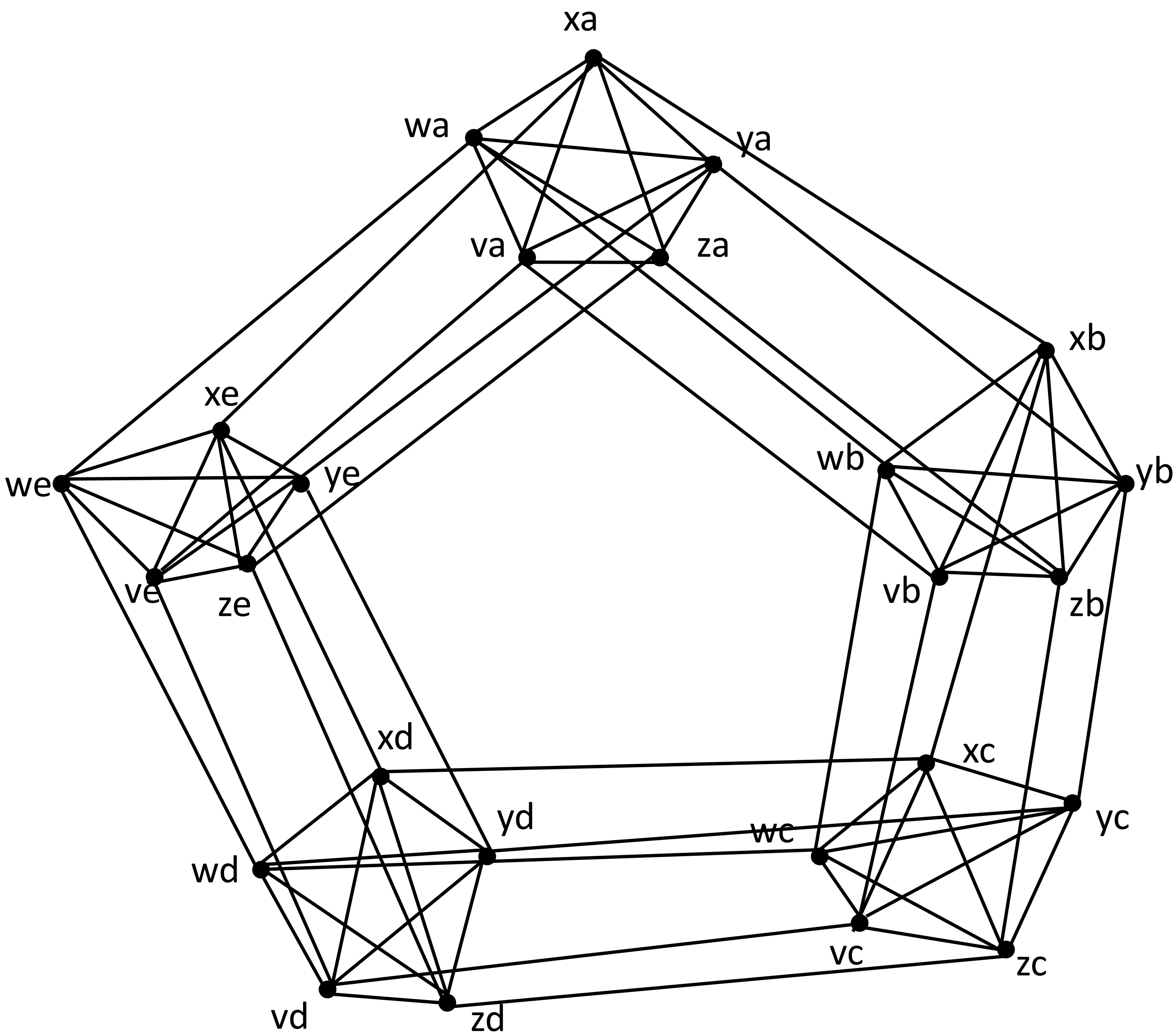

Figure 1.4

The detour distance between any two vertices is 24. The degree of each vertex is 6. All the vertices of the graph lie in the detour path. Therefore the detour M -distance of any two vertex of $K_5 \times K_5$ is 199.

Let us assume that the theorem is true for all cartesian product graph of K_{n-1} with $(n-1)^2$ vertices.

Now we prove the theorem for Cartesian product graph of K_n with n^2 vertices.

Let $K_n \times K_n$ is the Cartesian product of complete graph K_n. It has n^2 vertices. Every vertex has $(n+1)$ degree. All the vertices lie in the detour path. Therefore the detour M -distance between any two vertices of $K_n \times K_n$ is the sum of detour distance of any two vertex, degrees of all vertices in the detour path and number of vertices in the detour path and number of vertices in the path. Hence the detour M -distance of any two vertices of $K_n \times K_n$ graph is $n^3 + 3n^2 - 1$.

Hence the result.

1.3 Theorem

Let L_n be a ladder graph. The detour M -distance of cartesian product of the ladder graph L_n is $32n^2 - 16n - 1$.

Proof:

Let us prove the theorem by induction on the number of vertices in $L_n \times L_n$.

Let $n = 3$, then $L_3 \times L_3$ is the Cartesian product of ladder graph L_3 with 36 vertices. The Cartesian product of the graph L_3 will be of the form.

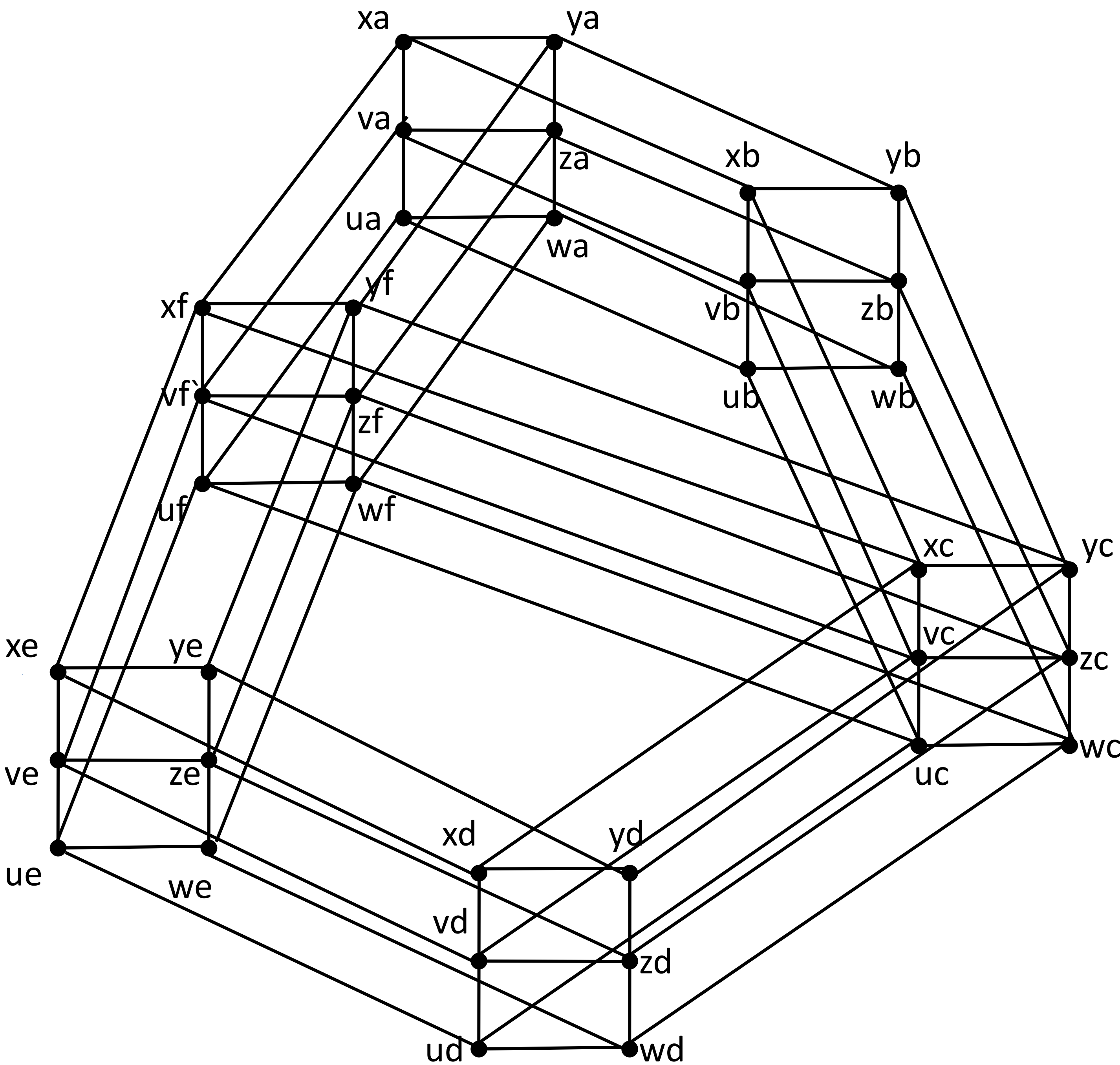

Figure 1.5

The above graph is a connected graph. The detour distance between any two vertices of the graph is 35.In $L_3 \times L_3$ graph is,16 vertices has degree 4, 16 vertices has degree 5, 4 vertex of the above graph has degree 6. All the vertices lie in the detour path of the graph.

The detour M distance of the Cartesian product of the graph L_3 is

$$D^M\left(L_n \times L_n\right) = D(u,v) + \sum_{W \in D(u,v)} \deg(W) + \sum_{W \in D(u,v)} |W|$$

$$D^M\left(L_3 \times L_3\right) = 239$$

Hence the detour M distance of $L_3 \times L_3$ is 239.

Let $n = 4$, then $L_4 \times L_4$ is the Cartesian product of ladder graph L_4 with 64 vertices.

The Cartesian product of ladder graph L_4 is of the form.

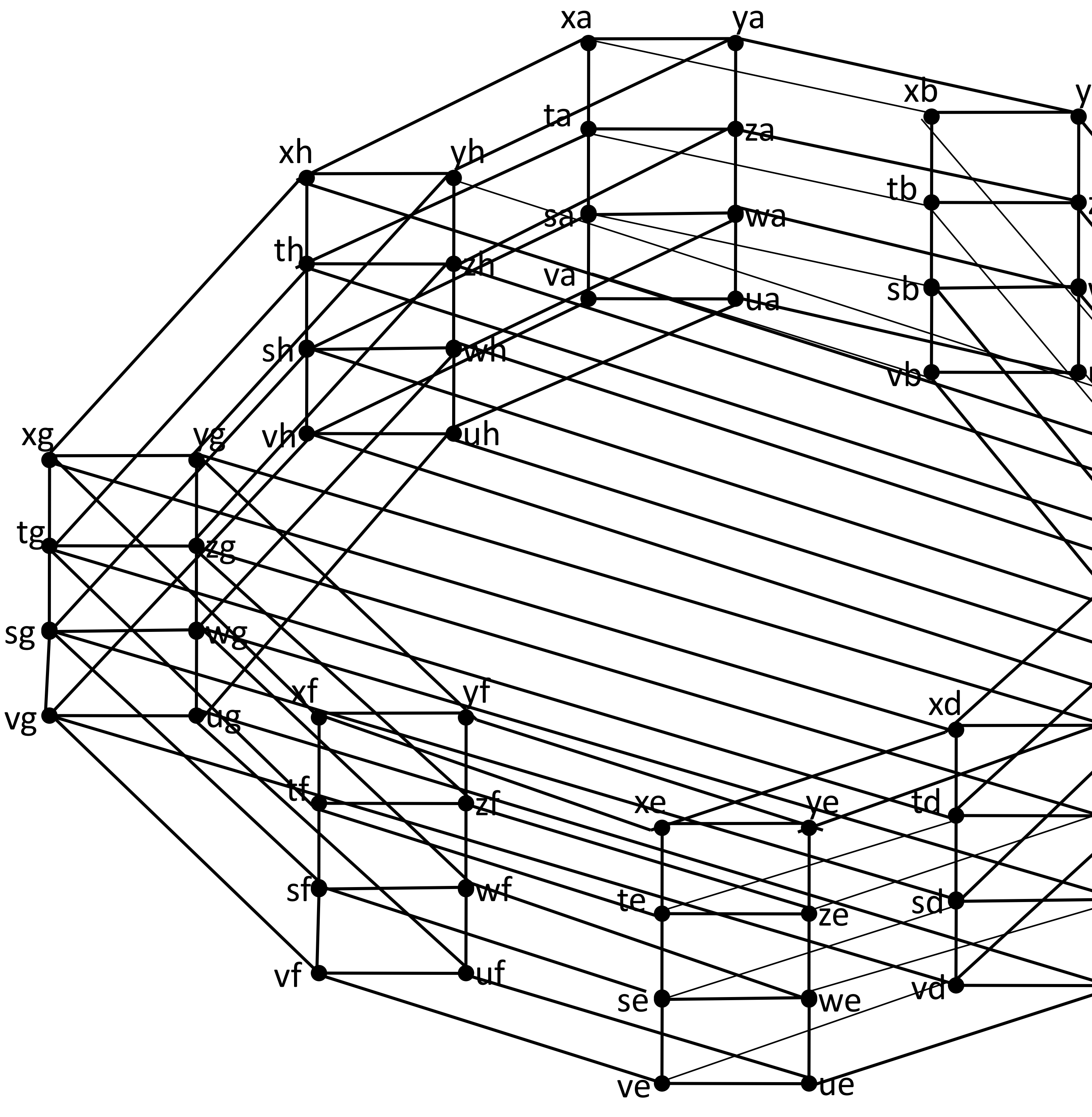

Figure 1.6

The detour distance between any two vertices of the graph is 63. In $L_4 \times L_4$ graph is 16 vertices has degree 4, 32 vertices has degree 5, 16 vertex of the above graph has degree 6.

Hence the detour M -distance of $L_4 \times L_4$ is 447.

Let us assume that the theorem is true for all Cartesian product of the graph L_{n-1} with $(n-1)^2$ vertices.

Now we prove the theorem for Cartesian product of the graph L_n with n^2 vertices.

Let L_n is the Ladder graph. $L_n \times L_n$ be the Cartesian product of the ladder graph L_n. It has n^2 vertices. the detour distance between any two vertices of $L_n \times L_n$ is $n^2 - 1$.

Hence the detour M -distance between any two vertex of $L_n \times L_n$ graph is $32n^2 - 16n - 1$.

Hence the result.

1.4 Theorem

Let S_n be a star graph. The detour M -distance of cartesian product of the star graph S_n is $2n^2 + 12n - 13$.

Proof:

Let us prove the theorem by induction on the number of vertices in $S_n \times S_n$.

Let $n = 3$, then $S_3 \times S_3$ is the Cartesian product of star graph S_3 with 9 vertices. The Cartesian product of the graph S_3 will be of the form.

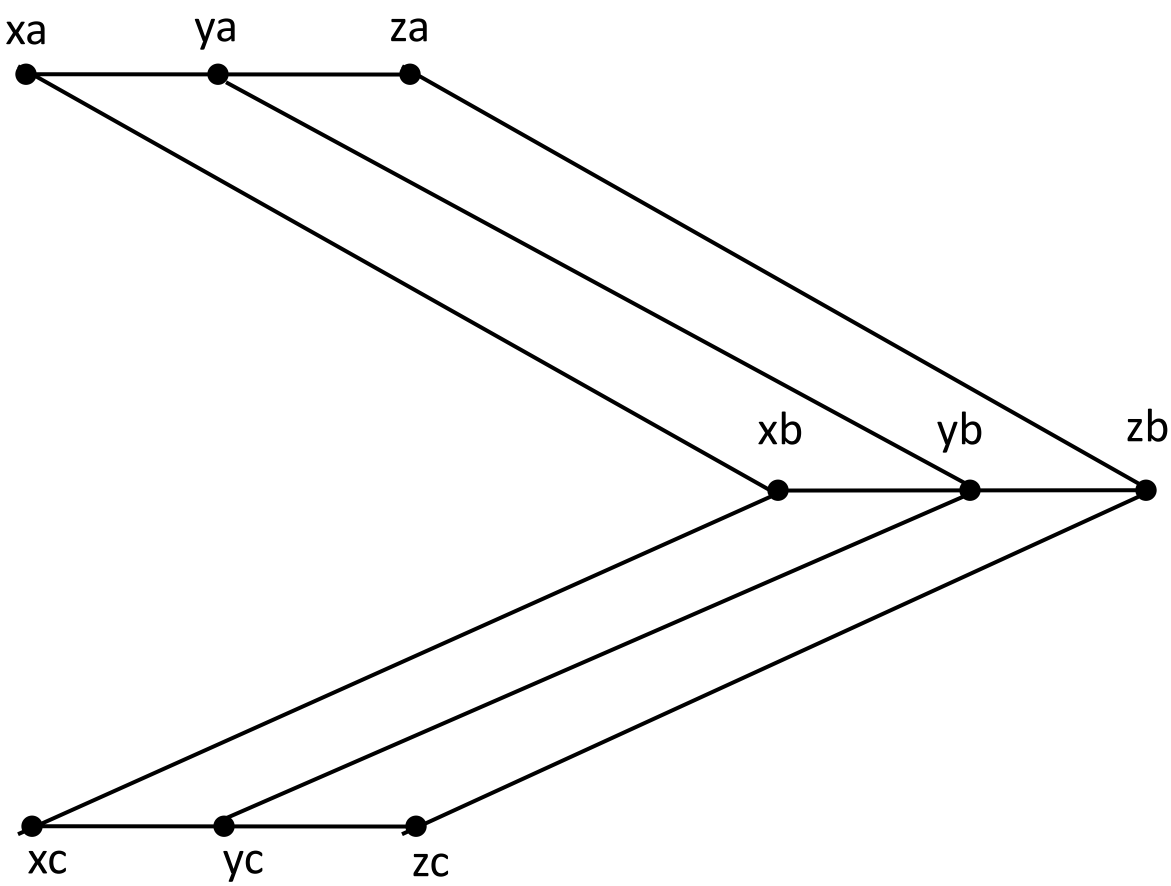

Figure 1.7

The above graph is a 4- regular graph. The detour distance between any two vertices of the graph is 8.In $S_3 \times S_3$ graph is 4 vertices has degree 2, 4 vertices has degree 3, 1 vertex of the above graph has degree 4. All the vertices lie in the detour path of the graph.

The detour M distance of the Cartesian product of the graph S_3 is

$$D^M(S_n \times S_n) = D(u,v) + \sum_{W \in D(u,v)} \deg(W) + \sum_{W \in D(u,v)} |W|$$

$$D^M(S_3 \times S_3) = 41$$

ie) $D^M(S_3 \times S_3) = 2(3)^2 + 12(3) - 13 = 41$

Hence the detour M distance of $S_3 \times S_3$ is 41.

Let $n = 4,$ then $S_4 \times S_4$ is the Cartesian product of star graph S_4 with 16 vertices.

The Cartesian product of star graph S_4 is of the form.

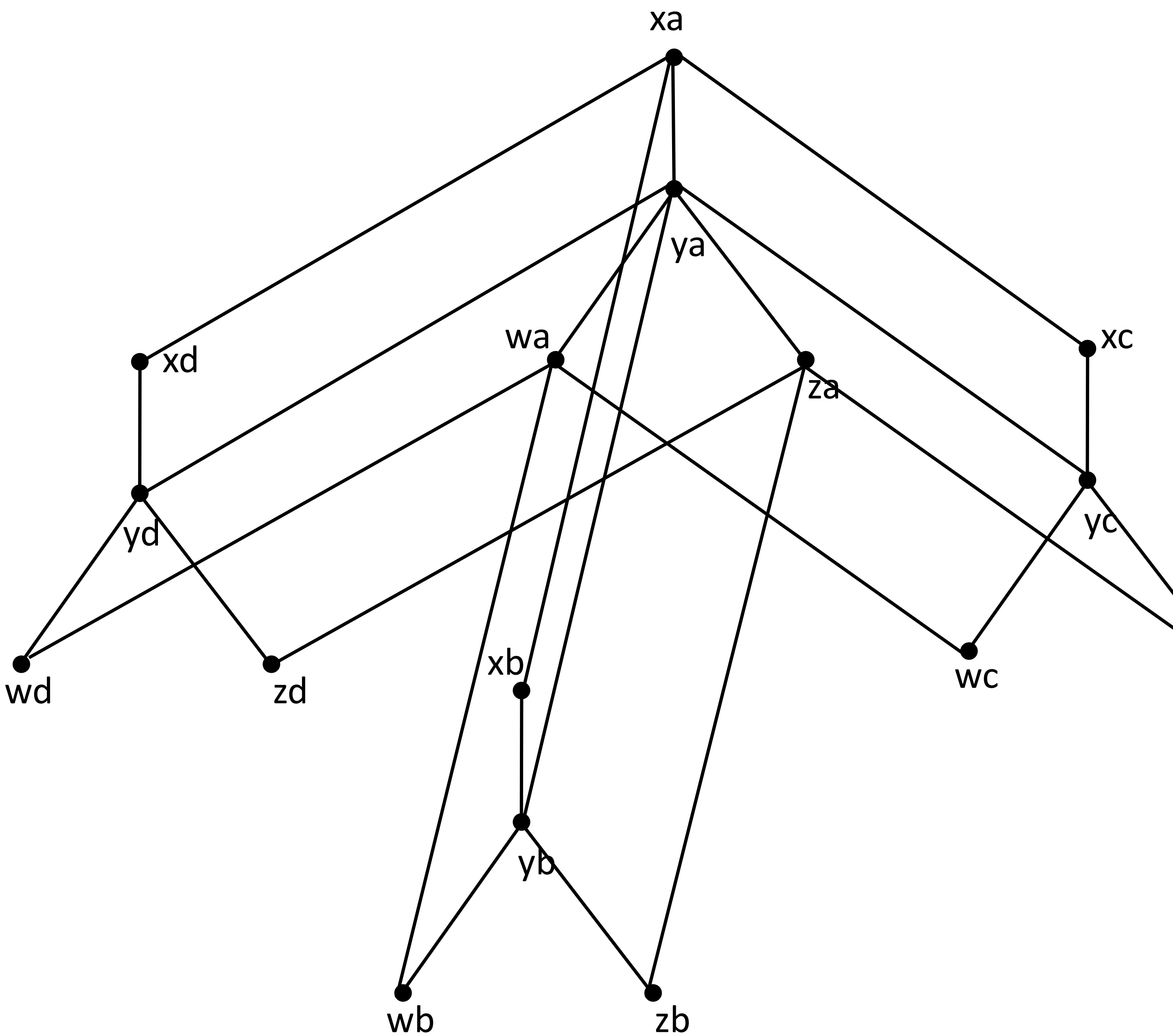

Figure 1.8

The Cartesian product of the Star graph is also 4 regular graph. The detour distance between any two vertices of the graph is 12. In $S_4 \times S_4$ graph is 6 vertices has degree 2, 6 vertices has degree 4, 1 vertex of the above graph has degree 6.

Hence the detour M -distance of $S_4 \times S_4$ is 67.

Let us assume that the theorem is true for all Cartesian product of the graph S_{n-1} with $(n-1)^2$ vertices.

Now we prove the theorem for Cartesian product of the graph S_n with n^2 vertices.

Let S_n is the Star graph. $S_n \times S_n$ be the Cartesian product of the star graph S_n. It has n^2 vertices. This graph is also 4-rergular graph. The detour distance between any two vertices of $S_n \times S_n$ is $n^2 - 1$.

Hence the detour M -distance between any two vertex of $S_n \times S_n$ graph is $2n^2 + 12n - 13$.

Hence the result.

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