Domination Number To the Kronecker Product of Star Graph with its Transformation

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Abstract : Let G_1 and G_2 be two graphs.The Kronecker product G_1 (K) G_2 has vertex set $V(G_1 (K) G_2) = V(G_1) \times V(G_2)$ and edge set $E(G_1 (K) G_2) = \{(u_1, v_1)(u_2, v_2) / u_1u_2 \in E(G_1), v_1v_2 \in E(G_2)\}$. In this paper, we have found the domination number and the dominating sets to Kronecker product of $K_{1,n}$ with its transformation graphs G^{+++} , G^{++-} , G^{+-+} , G^{-++} and G^{--+} . Also we have discussed some results.

Keywords : Kronecker product, domination number, dominating set, transformation, star graph.

INTRODUCTION

Domination is one of the interesting research area in graph theory. Wu and Meng have studied the concept of graph transformation and many applications have been studied in this topic.A graph G consists of a pair (V(G),E(G)) where V(G) is a non empty finite set whose elements are called vertices and E(G) is a set of unordered pairs of distinct elements of V(G). A graph that contains no cycles is called an acyclic graph. A connected acyclic graph is called a tree.

For S ⊆ V, if every vertex of V is either an element of S or V-S is said to be a dominating set and the corresponding dominating set is called a γ-set of G. The open neighborhood $N(v)$ of $v \in V$ is the set of vertices adjacent to v, that is, $N(v) = \{u / uv \in E(G)\}$ and the closed neighborhood of v is $N[v] = N(v) \cup \{v\}$.

Let $G = \big(V(G), E(G)\big)$ be a graph and x, y,z be three variables taking values + or -. The **transformation graph G^{xyz}** is the graph having $V(G) \cup E(G)$ as the vertex set and for $\propto, \beta \in V(G) \cup E(G)$, $\propto$ *and* β are adjacent in G^{xyz} if and only if one of the following holds:

(i) $\propto, \beta \in V(G)$. $\propto$ *and* β are adjacent in G if x = +; $\propto$ *and* β are not adjacent in G if x = - .

(ii) $\propto, \beta \in E(G). \propto \ and \ \beta$ are adjacent in G if y = +; $\propto \ and \ \beta$ are not adjacent in G if y = - .

(iii) $\propto \in V(G), \beta \in E(G). \propto \ and \ \beta$ are incidentt in G if z = +; $\propto \ and \ \beta$ are not incident in G if z = - .

MAIN RESULTS

Theorem: 1

Let $G^* = G\ (K)\ G^{+++}$ where $G = K_{1,n}$, then $\gamma(G^*) = 3$ and $|D(G^*)| = 2n^2$.

Proof:

Let $G = K_{1,n}$ and G^{+++} be a transformation of G.

Let $V(G) = \{u_i / \ 0 \le i \le n\}$ with $d(u_0) = n$ and $d(u_i) = 1$ for all $1 \le i \le n$ and $u_0 u_i = e_i / \ 1 \le i \le n$.

$V(G^{+++}) = \{v_i, e_j / \ 0 \le i \le n, 1 \le j \le n\}$ with $d(v_0) = 2n$, $d(v_i) = 2$ for all $1 \le i \le n$ and $d(e_j) = n$+1 for all $1 \le i \le n$.

$G^* = G\ (K)\ G^{+++}$ then $V(G^*) = \{u_i v_j, u_i e_j / \ 1 \le i, j \le n;\ 1 \le k \le n\}$ with $d(u_0 v_0) = 2n^2$,

$d(u_0 v_j) = 2n, 1 \le j \le n,\ d(u_0 e_j) = n(n+2) / \ 1 \le j \le n\ , d(u_i v_j) = 2,\ 1 \le i, j \le n$.

Also, $N(u_0 v_0) = \{u_i v_j, u_i e_j / \ i \ne 0\ \&\ j \ne 0\}\ , 1 \le i \le n, 1 \le j \le n$

$N(u_i v_0) = \{u_0 v_j, u_0 e_j / \ \ j \ne 0, 1 \le j \le n\}\ , i \ne 0$ and $1 \le i \le n$.

Also, $\{u_i v_0 / \ 1 \le i \le n\} \subset N(u_i v_0)$ and $\{u_i v_0 / \ 1 \le i \le n\} \in N(u_0 e_i)$.

Let $D_1 = \{\{u_0 v_0, u_i v_0, u_0 v_i\} / \ 1 \le i \le n\}$

$D_2 = \{\{u_0 v_0, u_i v_0, u_0 e_i\} / \ 1 \le i \le n\}$

Then, $N[u_0 v_0, u_i v_0, u_0 v_i] = V(G^*)$ for all $1 \le i \le n$

$N[u_0 v_0, u_i v_0, u_0 e_i] = V(G^*)$ for all $1 \le i \le n$

Hence, each set of D_1 and D_2 is a dominating set of G^*.

$\Longrightarrow \gamma(G^*) = 3$ and $|D(G^*)| = n^2 + n^2 = 2n^2$.

Result: 2

If $G = K_{1,n}$, then

i) $\gamma(G^*) > \gamma(G)\ (K)\ \gamma(G^{+++})$
ii) $\gamma(G^*) > \gamma(G) + \gamma(G^{+++})$

From the graph G,we know that $\gamma(G) = 1$ and $\gamma(G^{+++}) = 1$

Hence the result is trivial.

Result : 3

The vertices u_0v_0 , u_iv_0 and u_0v_i are not connected in G^*. Hence D is not a connected dominating set of G^*.

Theorem: 4

Let $G = K_{1,n}$ and G^{++-} is a transformation of G then $\gamma(G^*) = 4$; where $G^* = G\ (K)\ G^{+++}$

Proof:

Let $G = K_{1,n}$, $V(G) = \{u_0 / \ 0 \le i \le n\}$

Let $V(G^{++-}) = \{v_0, v_i, e_i / \ 1 \le i \le n\}$

Let $G^* = G\ (K)\ G^{++-}$ and $V(\ G^*) = \{u_iv_j, u_ie_k / \ 0 \le i, j \le n, 1 \le k \le n\}$

Clearly, N $(u_0v_0) = \{u_iv_j / \ 1 \le i, j \le n\}$

$N(u_0e_i) = V(G^*) - \{u_0v_j, u_0e_j, u_je_i, u_jv_i, u_jv_0, u_0v_0 / \ 1 \le j \le n\}$ for any fixed i.

For any element (u_je_i) , $j \neq 0$

$N(u_je_i) = \{u_0e_j / \ 1 \le j \le n$ and $i \neq j\} \cup \{u_0v_j / \ 1 \le j \le n, i \neq j\}$for any fixed i.

$N(u_0v_i) = \{u_jv_0 / \ 1 \le j \le n\}$

Also, $\{u_jv_i / \ 1 \le j \le n\} \subseteq N(u_0v_0)$ for any fixed i.

$\Longrightarrow N[u_0e_i, u_je_i, u_0v_i, u_0v_0] = V(G^*)$ for any fixed i and $j = 1,2,3, \dots, n$

Hence any set containing four elements of the form $\{u_0e_i, u_je_i, u_0v_i, u_0v_0\}$ is a dominating set of G^* . Hence $D_i = \{u_0e_i, u_je_i, u_0v_i, u_0v_0\}$ $1 \le j \le n$ are the dominating sets of G^*.

$\Longrightarrow \gamma(G^*) = 4$

Result:5

If $G^* = G\ (K)\ G^{++-}$ where$G = K_{1,n}$. Then$\gamma(G^*) > \gamma(G).\gamma(G^{++-})$.

Proof:

Let $G = K_{1,n}$, $V(G) = \{u_i / 1 \leq i \leq n\}$ with $d(u_0) = n$ and $d(u_i) = 1$ for all $1 \leq i \leq n$

$V(G^{++-}) = \{v_i, e_j / 0 \leq i \leq n, 1 \leq j \leq n\}$ with $d(v_i) = n, 0 \leq i \leq n$ and

$d(e_i) = 2(n-1), 1 \leq i \leq n.$

In G, $N[v_0] = V(G) \implies \gamma(G) = 1$

In G^{++-}, $N[v_0, e_i] = N[v_i, e_i] = V(G^{++-})$ for all $i = 1,2,3, \dots, n$

$\implies \gamma(G^{++-}) = 2$

By theorem : 4, $\gamma(G^*) = 4$

$\implies \gamma(G^*) > \gamma(G). \gamma(G^{++-}).$

Hence the result.

Theorem:6

If $G^* = G\ (K)\ G^{+-+}$ where $G = K_{1,n}$ then $\gamma(G^*) = 3$ and $|D| = 2n^2$.

Proof:

Let $G = K_{1,n}$, then $V(G) = \{u_i / 0 \leq i \leq n\}$

$d(u_0) = n$; $d(u_i) = 1$ for all $i = 1,2, \dots, n$; $e_i = u_0 u_i$, $1 \leq i \leq n$

In G^{+-+}, $V(G) = \{v_i, e_j / 0 \leq i \leq n; 1 \leq j \leq n\}$ with $d(v_0) = 2n$ and

$d(v_i) = d(e_i) = 2$ for all $i = 1,2, \dots, n$

Let $G^* = G(K)G^{+-+}$, then $V(G^*) = \{u_i v_j, u_i e_k / \quad 0 \leq i, j \leq n; 1 \leq k \leq n\}$ with $d(u_0 v_0) = 2n^2$; $d(u_0 v_i) = d(u_0 e_i) = d(u_i v_0) = 2n$.

Clearly $N(u_0 v_0) = \{u_i v_j, u_i e_j / 0 < i, j \leq n, i, j \neq 0\}$

$N(u_0 v_i) = \{u_j v_0 / 1 \leq j \leq n\}$ for all $i = 1,2, \dots, n$

$N(u_0 e_i) = \{u_j v_0 / 1 \leq j \leq n\}$ for all $i = 1,2, \dots, n$

$N(u_0 v_i) = \{u_0 v_j, u_0 e_j / 0 \leq j \leq n\}$ for all $1 \leq i \leq n$

Let $D_i = N[u_0 v_0, u_0 v_i, u_i v_0] = N[u_0 v_0, u_0 e_i, u_i v_0] = V(G^*)$ for all $1 \leq i \leq n$

Hence, $\gamma(G^*) = 3$ and $|D_i| = 2n^2$.

Result:7

$G^* = G\ (K)\ G^{+-+}$ where $G = K_{1,n}$ then $\gamma(G^*) > \gamma(G).\gamma(G^{+-+})$.

Proof:

We know that $G = K_{1,n}, then\ \gamma(G) = 1$

In G^{+-+}, $N[v_0] = V(G^{+-+})$

Hence, $\gamma(G^{+-+}) = 1$

$\gamma(G).\gamma(G^{+-+}) = 1$

By theorem, $\gamma(G^*) = 3$

$\Longrightarrow \gamma(G^*) > \gamma(G).\gamma(G^{+-+})$.

Theorem:8

If $G^* = G\ (K)\ G^{-++}$ where $G = K_{1,n}$ then $\gamma(G^*) = 5$.

Proof:

Let $G = K_{1,n}$ and $V(G) = \{u_i/\ 0 \le i \le n\}$ with $d(u_0) = n$ and $d(u_i) = 1$ for all $i = 1,2, \dots n$

$V(G^{-++}) = \{v_i, e_j/\ 0 \le i \le n; 1 \le j \le n\}$

Let $G^* = G\ (K)\ G^{-++}$ and then $V(G^*) = \{u_i v_j, u_i e_k/\ 0 \le i,j \le n; 1 \le k \le n\}$ with

$d(u_0 v_0) = n^2, d(u_0 v_i) = n^2,$

$d(u_i v_j) = d(u_i e_j) = n, 1 \le i \le n; 1 \le j \le n$

$d(u_0 e_j) = n(n+1)$ for all $1 \le j \le n$

Clearly, $N(u_0 v_0) = \{u_i e_j/\ 1 \le i,j \le n\}$

$N(u_0 v_i) = \{u_i v_j/\ 1 \le i \le n, 1 \le j \le n, i \ne j\}$

$N(u_j v_i) = \{u_o v_j/\ 1 \le j \le n, i \ne j\}$

$N(u_i v_0) = \{u_o e_j / 1 \leq j \leq n\}$ for all $i = 1,2, \ldots, n$

$N(u_0 e_i) = \{u_j v_0, u_j v_i / 1 \leq j \leq n\}$ for all $i = 1,2, \ldots, n$

Hence, the elements of the form,

$D_i = \{u_0 v_0, u_0 v_i, u_0 e_i, u_i v_0, u_i v_i\}$ is a dominating sest of G.

For example $\{u_0 v_0, u_0 v_1, u_0 e_1, u_1 v_0, u_1 v_1\}$ is a dominating set of G

$$\Longrightarrow \gamma(G^*) = 5.$$

Theorem:9

If $G^* = G\ (K) G^{--+}$ where $G = K_{1,n}$ then $\gamma(G^*) = 5$.

Proof:

Let $V(G) = \{u_i / 0 \leq i \leq n\}$, $E(G) = \{e_i = u_0 u_i, 0 \leq i \leq n\}$

$d(u_0) = n$ and $d(u_i) = 1$, $1 \leq i \leq n$

$V(G^{--+}) = \{v_i, e_j / 0 \leq i \leq n; 1 \leq j \leq n\}$, $d(v_i) = n$ for all $0 \leq i \leq n$; $d(e_i) = 2$ for all i.

Let $G^* = G\ (K)\ G^{--+}$ be the graph.

Then, $V(G^*) = \{u_i v_j, u_i e_j / 0 \leq i \leq n; 1 \leq j \leq n\}$ with $d(u_0 v_0) = n^2$;

$d(u_0 e_j) = 2n$; $d(u_i v_j) =$ n for all $0 \leq i \leq n; 1 \leq j \leq n$.

Let $S_{01} = \{u_0 v_j / 0 \leq j \leq n\}$

$S_{02} = \{u_0 e_j / 0 \leq j \leq n\}$

$S_{10} = \{u_j v_0 / 0 \leq j \leq n\}$

$S_{iv} = \{u_i v_j / 1 \leq j \leq n; 1 \leq i \leq n\}$

$S_{ie} = \{u_i e_j / 1 \leq j \leq n, 1 \leq i \leq n\}$

Also, $N(u_0 v_0) = S_{ie}$

$N(u_i v_0) = S_{02}$

$N(u_0 e_i) = \{u_j v_0, u_j v_i / 1 \leq i \leq n\}$

$S_{iv} \subseteq N(u_0 v_i, u_0 v_j)\ i \neq j$

$S_{10} \subseteq N(u_0 e_j), 1 \leq i \leq n$

$S_{02} \subseteq\ N(u_j v_0), 1 \leq j \leq n$

Hence, every collection of the form,

$D_i = \{u_0 v_0, u_i v_0, u_0 e_i, u_0 v_i, u_0 v_j / \ 1 \leq j \leq n\}$ for all i is a dominating set of G^*.

Hence, $\gamma(G^*) = 5$.

CONCLUSION

In this paper, we have found the domination number and the dominating sets to the Kronecker product of $K_{1,n}(K)G^{+++}$, $K_{1,n}(K)G^{++-}$, $K_{1,n}(K)G^{+-+}$, $K_{1,n}(K)G^{-++}$ and $K_{1,n}(K)G^{--+}$.

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