Upper Signed Edge Unidomination Number of a Rooted Product Graph of a Path with a Cycle

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ABSTRACT

Graph theory is a central branch of Mathematics. In current years it was developed exponentially. Domination in graphs is quickly emerging area of research in graph theory and it has become the foundation of interest of many researchers. The concept of Signed dominating function was introduced by Dunbar et al. [4]. Edge domination was introduced by Mitchell and Hedetneimi [8]. Bharathi [3] has introduced by new concept edge unidomination and studied this for complete K- partite graph.

Signed unidominating function was introduced by Aruna [1] in 2019 and studied this for some corona product graphs. Signed edge domination on rooted product graph studied by Shobha Rani [10]. The concept of signed unidomination number of a rooted product graph of a path with a cycle was studied by Durgavathi [5]. In this paper, we introduced signed edge unidominating function of a graph and determine signed edge unidomination number of a rooted product graph of a path with a cycle.

Keywords: Signed edge unidominating function, minmal signed edge unidominating function, signed edge unidomination number, upper signed edge unidomination number, rooted product graph, path and cycle.

1. INTRODUCTION

Currently, the major expansion of graph theory has occurred and motivated to a larger degree and has become the source of importance to many researchers due to its applications to various branches of Science & Technology.

An introduction and a wide summary on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [6, 7].The theory of domination in graphs introduced by Ore [9] and Berge [2] is a fascinated area of research in graph theory in the last three decades.

In this paper a new concept signed edge unidominating function of a graph is introduced and this is studied for a rooted product graph of a path with a cycle. Also signed edge unidomination number and upper signed edge unidomination number of this graph is found in different cases.

2. SIGNED EDGE UNIDOMINATING FUNCTION AND SIGNED EDGE UNIDOMINATION NUMBER

In this section, signed edge unidominating function and signed edge unidomination number of a graph are defined as follows:

Definition 1: Let $G(V,E)$ be a connected graph. A function $g: E \rightarrow \{-1,1\}$ is said to be a signed edge unidominating function, if

$$\sum_{e\prime \in N[e]} g(e') \geq 1 \quad \forall\, e \in E \text{ and } g(e) = 1,$$

$$\sum_{e\prime \in N[e]} g(e') = 1 \quad \forall\, e \in E \text{ and } g(e) = -1$$

where $N[e]$ is the closed neighbourhood of the edge e.

Definition 2: The signed edge unidomination number of a graph $G(V,E)$ is defined as

$$min\{g(E)/g \text{ is a signed edge unidominating function}\}.$$

It is denoted by $\gamma'_{su}(G)$.

Here $g(E) = \sum_{e\prime \in E} g(e')$ *is called as the weight of the signed edge unidominating function g.*

That is the signed edge unidomination number of a graph $G(V,E)$ is the minimum of the weights of the signed edge unidominating functions of G.

3. SIGNED EDGE UNIDOMINATION NUMBER OF ROOTED PRODUCT GRAPH OF A PATH WITH A CYCLE

In this section we discuss signed edge unidominating function of rooted product graph of a path with a cycle and signed edge unidomination number of this graph is obtained in various cases.

Theorem 3.1: The signed edge unidomination number of rooted product graph $P_n \circ C_m$ is

$$\begin{cases} \frac{1}{3}[mn + 3n - 3] & if \;\; m \equiv 0(mod\ 3) \\ \frac{1}{3}[mn + 5n - 3] & if \;\; m \equiv 1(mod\ 3) \\ \frac{1}{3}[mn + 7n - 3] & if \;\; m \equiv 2(mod\ 3) \end{cases}$$

Proof: Let $P_n \circ C_m$ be the given rooted product graph.

The edges in P_n are denoted by $e_1, e_2, \dots, e_{n-1}$ and the i^{th} copy of C_m are denoted by $e_{i1}, e_{i2}, \dots, e_{im-1}, e_{im}$.

The following three cases arise.

Case 1: Let $m \equiv 0(mod3)$.

Define a function $g: E \rightarrow \{-1,1\}$ by

$$g(e_i) = 1$$

and $g(e_{ij}) = \begin{cases} -1 & if\ j \equiv 2(mod\ 3), \\ 1 & otherwise \end{cases}$

for $i = 1,2, \dots \dots, n-1$ and $j = 1,2, \dots \dots, m$.

Let $i \neq 1$ and $i \neq n-1$.

If $e_i \in P_n$ then

$$\sum_{e' \in N[e_i]} g(e') = g(e_{i-1}) + g(e_i) + g(e_{i+1}) + g(e_{i1}) + g(e_{im}) + g(e_{i+1\,1}) + g(e_{i+1\,m})$$

$$= 1 + 1 + 1 + 1 + 1 + 1 + 1 = 7.$$

Let $i = 1$. Then

$$\sum_{e' \in N[e_1]} g(e') = g(e_1) + g(e_2) + g(e_{11}) + g(e_{1m}) + g(e_{2\,1}) + g(e_{2\,m}) = 1 + 1 + 1 + 1 + 1 + 1$$

$$= 6.$$

Same as we prove for $i = n-1$.

If $e_{ij} \in C_m$ then $g(e_{\mathrm{ij}}) = 1$ or $g(e_{\mathrm{ij}}) = -1$.

We have the following cases.

Sub Case 1: Let $j \equiv 0(mod\ 3)$. Then $g(e_{ij}) = 1$.

Let $j \neq m$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = (-1) + 1 + 1 = 1.$$

Let $j = m$. Then

$$\sum_{e' \in N[e_{im}]} g(e') = g(e_{im}) + g(e_{i\,m-1}) + g(e_{i1}) + g(e_i) + g(e_{i-1}) = 1 + (-1) + 1 + 1 + 1$$

$$= 3.$$

Sub Case 2: Let $j \equiv 1(mod\ 3)$. Then $g(e_{ij}) = 1$.

Let $j \neq 1$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = 1 + 1 + (-1) = 1.$$

Let $j = 1$. Then

$$\sum_{e' \in N[e_{i1}]} g(e') = g(e_{i1}) + g(e_{i2}) + g(e_{im}) + g(e_i) + g(e_{i-1}) = 1 + (-1) + 1 + 1 + 1$$
$$= 3.$$

Sub Case 3: Let $j \equiv 2(mod\ 3)$. Then $g(e_{ij}) = -1$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = 1 + (-1) + 1 = 1.$$

That is g is satisfying the conditions of a signed edge unidominating function and hence it follows that g is a signed edge unidominating function.

$$\text{Now } g(E) = \sum_{e' \in P_n} g(e') + \sum_{e' \in C_m} g(e')$$

$$= \underbrace{(1 + 1 + 1 \ldots + 1)}_{(n-1)-times} + \underbrace{\underbrace{\{(1 + (-1) + 1) + (1 + (-1) + 1) + \cdots + (1 + (-1) + 1)\}}_{\left(\frac{m}{3}\right)-\text{times}}}_{n-times}$$

$$= n - 1 + \left(\frac{m}{3}\right) n = \frac{1}{3}[mn + 3n - 3].$$

Thus $g(E) = \frac{1}{3}[mn + 3n - 3]$.

Now for all other possibilities of assigning values 1 and -1 to the edges of P_n and

edges e_{ij} in each copy of C_m , we can show that the resulting functions are not

signed edge unidominating functions.

Hence the function defined above is the only signed edge unidominating function.

Therefore $\gamma'_{su}(P_n \circ C_m) = \frac{1}{3}[mn + 3n - 3]$ when $m \equiv 0(mod\ 3)$.

Case 2: Let $m \equiv 1(mod 3)$.

Define a function $g: E \rightarrow \{-1,1\}$ by

$g(e_i) = 1$

and $g(e_{ij}) = \begin{cases} -1 & if\ j \equiv 2(mod\ 3), \\ 1 & otherwise \end{cases}$

for $i = 1,2, \dots \dots, n-1$ and $j = 1,2, \dots \dots, m$.

We prove same as above case for $e_i \in P_n$.

If $e_{ij} \in C_m$ then $g(e_{ij}) = 1$ or $g(e_{ij}) = -1$.

We have the following cases.

Sub Case 1: Let $j \equiv 0(mod\ 3)$. Then $g(e_{ij}) = 1$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = (-1) + 1 + 1 = 1.$$

Sub Case 2: Let $j \equiv 1(mod\ 3)$. Then $g(e_{ij}) = 1$.

Let $j \neq 1$ and $j \neq m$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = 1 + 1 + (-1) = 1.$$

Let $j = 1$. Then

$$\sum_{e' \in N[e_{i1}]} g(e') = g(e_{i1}) + g(e_{i2}) + g(e_{im}) + g(e_i) + g(e_{i-1}) = 1 + (-1) + 1 + 1 + 1$$
$$= 3.$$

Let $j = m$. Then

$$\sum_{e' \in N[e_{im}]} g(e') = g(e_{im}) + g(e_{i\,m-1}) + g(e_{i1}) + g(e_i) + g(e_{i-1}) = 1 + 1 + 1 + 1 + 1$$
$$= 5.$$

Sub Case 3: Let $j \equiv 2(mod\ 3)$. Then $g(e_{ij}) = -1$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = 1 + (-1) + 1 = 1.$$

That is g is satisfying the conditions of a signed edge unidominating function and hence it follows that g is a signed edge unidominating function.

$$\text{Now } g(E) = \sum_{e' \in P_n} g(e') + \sum_{e' \in C_m} g(e')$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-1)-times} +$$

$$\underbrace{\underbrace{\{(1+(-1)+1)+(1+(-1)+1)+\cdots+(1+(-1)+1)\}}_{\left(\frac{m-1}{3}\right)-\text{times}} \quad +1}_{n-times}$$

$$= n - 1 + \left(\frac{m-1}{3}\right)n + n$$

$$= n - 1 + \left(\frac{m+2}{3}\right)n = \frac{1}{3}[mn + 5n - 3].$$

Thus $g(E) = \frac{1}{3}[mn + 5n - 3]$.

Now for all other possibilities of assigning values 1 and -1 to the edges of P_n and edges e_{ij} in each copy of C_m , we can show that the resulting functions are not signed edge unidominating functions.

Hence the function defined above is the only signed edge unidominating function.

Therefore $\gamma'_{su}(P_n \circ C_m) = \frac{1}{3}[mn + 5n - 3]$ when $m \equiv 1 (mod\ 3)$.

Case 3: Let $m \equiv 2(mod 3)$.

Define a function $g: E \to \{-1,1\}$ by

$$g(e_i) = 1$$

and $g(e_{ij}) = \begin{cases} -1 & if\ j \equiv 2(mod\ 3),\ j \neq m \\ 1 & otherwise \end{cases}$

for $i = 1,2, \ldots \ldots, n-1$ and $j = 1,2, \ldots \ldots, m$.

We prove same as above case for $e_i \in P_n$.

If $e_{ij} \in C_m$ then $g(e_{\mathrm{ij}}) = 1$ or $g(e_{\mathrm{ij}}) = -1$.

We have the following cases.

Sub Case 1: Let $j \equiv 0(mod\ 3)$. Then $g(e_{ij}) = 1$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = (-1) + 1 + 1 = 1.$$

Sub Case 2: Let $j \equiv 1(mod\ 3)$. Then $g(e_{ij}) = 1$.

Let $j \neq 1$ and $j \neq m - 1$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = 1 + 1 + (-1) = 1.$$

Let $j = 1$. Then

$$\sum_{e' \in N[e_{i1}]} g(e') = g(e_{i1}) + g(e_{i2}) + g(e_{im}) + g(e_i) + g(e_{i-1}) = 1 + (-1) + 1 + 1 + 1$$
$$= 3.$$

Let $j = m - 1$. Then

$$\sum_{e' \in N[e_{i\,m-1}]} g(e') = g(e_{i\,m-2}) + g(e_{i\,m-1}) + g(e_{im}) = 1 + 1 + 1 = 3.$$

Sub Case 3: Let $j \equiv 2(mod\ 3)$.

If $j \neq m$ then $g(e_{ij}) = -1$.

$$\sum_{e' \in N[e_{ij}]} g(e') = g(e_{ij-1}) + g(e_{ij}) + g(e_{ij+1}) = 1 + (-1) + 1 = 1.$$

If $j = m$ then $g(e_{ij}) = 1$.

$$\sum_{e' \in N[e_{im}]} g(e') = g(e_{im}) + g(e_{i\,m-1}) + g(e_{i1}) + g(e_i) + g(e_{i-1}) = 1 + 1 + 1 + 1 + 1$$
$$= 5.$$

That is g is satisfying the conditions of a signed edge unidominating function and hence it follows that g is a signed edge unidominating function.

$$\text{Now } g(E) = \sum_{e' \in P_n} g(e') + \sum_{e' \in C_m} g(e')$$

$$= \underbrace{(1 + 1 + 1 \ldots + 1)}_{(n-1)-times} +$$

$$\underbrace{\underbrace{\{(1+(-1)+1)+(1+(-1)+1)+\cdots+(1+(-1)+1)\}}_{\left(\frac{m-2}{3}\right)-\text{times}} +1+1}_{n-times}$$

$$= n-1+\left(\frac{m-2}{3}\right)n+2n$$

$$= n-1+\left(\frac{m+4}{3}\right)n = \frac{1}{3}[mn+7n-3].$$

Thus $g(E) = \frac{1}{3}[mn+7n-3]$.

Now for all other possibilities of assigning values 1 and -1 to the edges of P_n and

edges e_{ij} in each copy of C_m , we can show that the resulting functions are not

signed edge unidominating functions.

Hence the function defined above is the only signed edge unidominating function.

Therefore $\gamma'_{su}(P_n{\circ}\, C_m) = \frac{1}{3}[mn+7n-3]$ when $m \equiv 2(mod\ 3)$.

Theorem 3.2: For $m \equiv 0(mod\ 3)$, $m \equiv 1(mod\ 3)$, $m \equiv 2(mod\ 3)$ the number of signed edge unidominating functions of $P_n{\circ}\, C_m$ is 1 with minimum weights $\frac{1}{3}[mn+3n-3]$, $\frac{1}{3}[mn+5n-3]$, $\frac{1}{3}[mn+7n-3]$ respectively.

Proof: Follows by Theorem 3.1.

4. UPPER SIGNED EDGE UNIDOMINATION NUMBER OF ROOTED PRODUCT GRAPH

In this section the concepts of minimal signed edge unidominating function and upper signed edge unidomination number are defined as follows:

Definition 1: Let $G(V,E)$ be a graph and g and h be functions from E to $\{-1,1\}$. We say that $h < g$ if $h(e) \leq g(e)\ \forall\, e \in E$, with strict inequality for at least one edge e.

Definition 2: Let $G(V,E)$ be a connected graph. A signed edge unidominating function

$g: E \to \{-1,1\}$ is called a **minimal signed edge unidominating function** if for all $h < g$, h is not a signed edge unidominating function.

Definition 3: The **upper signed edge unidomination number** of a graph $G(V,E)$ is defined as

$\max\{g(E)/g \text{ is a minimal signed edge unidominating function}\}$.

It is denoted by $\Gamma'_{su}(G)$.

5. UPPER SIGNED EDGE UNIDOMINATION NUMBER OF ROOTED PRODUCT GRAPH OF A PATH WITH A CYCLE

In this section we discuss minimal signed edge unidominating function of rooted product graph of a path with a cycle and upper signed edge unidomination number of this graph is obtained in various cases.

Theorem 5.1: The upper signed edge unidomination number of rooted product graph $P_n{}^\circ C_m$ is

$$\begin{cases} \frac{1}{3}[mn+3n-3] & if \ \ m \equiv 0(mod\ 3) \\ \frac{1}{3}[mn+5n-3] & if \ \ m \equiv 1(mod\ 3) \\ \frac{1}{3}[mn+107n-3] & if \ \ m \equiv 2(mod\ 3) \end{cases}$$

Proof: Let $P_n{}^\circ C_m$ be the given rooted product graph.

Case 1: Let $m \equiv 0(mod 3)$.

Define a function $g: E \to \{-1,1\}$ by

$$g(e_i) = 1$$

and $g(e_{ij}) = \begin{cases} -1 & if\ j \equiv\ 2(mod\ 3)\ , \\ 1 & otherwise \end{cases}$

This function is same as the function defined in Case 1 of Theorem 3.1 and it is shown that g is a signed edge unidominating function.

Now we check for the minimality of g.

Define a function $h: E \to \{-1,1\}$ by

$$h(e_i) = \begin{cases} -1 & for\ e_i =\ e_k \in P_n\ for\ some\ k, where\ k = i \\ 1 & otherwise \end{cases}$$

and $g(e_{ij}) = \begin{cases} -1 & if\ j \equiv\ 2(mod\ 3)\ , \\ 1 & otherwise \end{cases}$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$.

Suppose $i = k$. Then $h(e_k) = -1$.

For $v_k \in P_n$ we have

$$\sum_{e' \in N[e_k]} g(e') = g(e_{k-1}) + g(e_k) + g(e_{k+1}) + g(e_{k1}) + g(e_{km}) + g(e_{k+1\,1}) + g(e_{k+1\,m})$$

$$= 1 + (-1) + 1 + 1 + 1 + 1 + 1 = 5 \neq 1.$$

This is the case when a signed edge unidominating function fails an edge $e_k \in P_n$ where $h(e_k) = -1$ because it is in the vicinity of the edge.

Therefore h is not a signed edge unidominating function.

Since h is defined arbitrarily, there is no $h < g$ such that h is a signed edge unidominating function.

As a result, g is a minimal signed edge unidominating function.

g is the only minimal signed edge unidominating function because assigning the functional values $-1, 1$ to the edges of P_n and C_m in any other way does not make g any longer a signed edge unidominating function.

$$\text{Now } g(E) = \sum_{e' \in P_n} g(e') + \sum_{e' \in C_m} g(e')$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-1)-times} + \underbrace{\underbrace{\{(1+(-1)+1)+(1+(-1)+1)+\cdots+(1+(-1)+1)\}}_{\left(\frac{m}{3}\right)-\text{times}}}_{n-times}$$

$$= n - 1 + \left(\frac{m}{3}\right)n = \frac{1}{3}[mn + 3n - 3].$$

Thus $g(E) = \frac{1}{3}[mn + 3n - 3]$.

Now max $\{g(E)/g$ is a minimal signed edge unidominating function$\} = \frac{1}{3}[mn + 3n - 3]$, because g is the only one minimal signed edge unidominating function.

Therefore $\Gamma'_{su}(P_n \circ C_m) = \frac{1}{3}[mn + 3n - 3]$ when $m \equiv 0(mod\ 3)$.

Case 2: Let $m \equiv 1(mod 3)$.

Define a function $g: E \to \{-1,1\}$ by

$$g(e_i) = 1$$

$$\text{and } g(e_{ij}) = \begin{cases} -1 & if\ j \equiv 2(mod\ 3), \\ 1 & otherwise \end{cases}$$

This function is same as the function defined in Case 2 of Theorem 3.1 and it is shown that g is a signed edge unidominating function.

Now we check for the minimality of g.

Define a function $h: E \to \{-1,1\}$ by

$$h(e_i) = \begin{cases} -1 & for\ e_i = e_k \in P_n\ for\ some\ k, where\ k = i \\ 1 & otherwise \end{cases}$$

$$\text{and } g(e_{ij}) = \begin{cases} -1 & if\ j \equiv 2(mod\ 3), \\ 1 & otherwise \end{cases}$$

for $i = 1,2, \ldots \ldots, n$ and $j = 1,2, \ldots \ldots, m$.

Suppose $i = k$. Then $h(e_k) = -1$.

For $v_k \in P_n$ we have

$$\sum_{e' \in N[e_k]} g(e') = g(e_{k-1}) + g(e_k) + g(e_{k+1}) + g(e_{k1}) + g(e_{km}) + g(e_{k+1\,1}) + g(e_{k+1\,m})$$

$$= 1 + (-1) + 1 + 1 + 1 + 1 + 1 = 5 \neq 1.$$

This is the case when a signed edge unidominating function fails an edge $e_k \in P_n$ where $h(e_k) = -1$ because it is in the vicinity of the edge.

Therefore h is not a signed edge unidominating function.

Since h is defined arbitrarily, there is no $h < g$ such that h is a signed edge unidominating function.
As a result, g is a minimal signed edge unidominating function.
g is the only minimal signed edge unidominating function because assigning the functional values $-1, 1$ to the edges of P_n and C_m in any other way does not make g any longer a signed edge unidominating function.

$$\text{Now } g(E) = \sum_{e' \in P_n} g(e') + \sum_{e' \in C_m} g(e')$$

$$= \underbrace{(1 + 1 + 1 \ldots + 1)}_{(n-1)-times} +$$

$$\underbrace{\underbrace{\{(1 + (-1) + 1) + (1 + (-1) + 1) + \cdots + (1 + (-1) + 1)\}}_{\left(\frac{m-1}{3}\right)-\text{times}} + 1}_{n-times}$$

$$= n - 1 + \left(\frac{m+2}{3}\right) n = \frac{1}{3}[mn + 5n - 3].$$

Thus $g(E) = \frac{1}{3}[mn + 5n - 3]$.

Now max $\{g(E)/g$ is a minimal signed edge unidominating function$\} = \frac{1}{3}[mn + 5n - 3]$, because g is the only one minimal signed edge unidominating function.
Therefore $\Gamma'_{su}(P_n \circ C_m) = \frac{1}{3}[mn + 5n - 3]$ when $m \equiv 1(mod\ 3)$.
Case 3: Let $m \equiv 2(mod 3)$.

Define a function $g: E \to \{-1,1\}$ by

$$g(e_i) = 1$$

$$\text{and } g(e_{ij}) = \begin{cases} -1 & if\ j \equiv 2(mod\ 3),\ j \neq m \\ 1 & otherwise \end{cases}$$

This function is same as the function defined in Case 3 of Theorem 3.1 and it is shown that g is a signed edge unidominating function.

Now we check for the minimality of g.

Define a function $h: E \to \{-1,1\}$ by

$$h(e_i) = \begin{cases} -1 & for\ e_i = e_k \in P_n\ for\ some\ k, where\ k = i \\ 1 & otherwise \end{cases}$$

and $$g(e_{ij}) = \begin{cases} -1 & if\ j \equiv 2(mod\ 3), \\ 1 & otherwise \end{cases}$$

for $i = 1,2, \dots\dots, n$ and $j = 1,2, \dots\dots, m$.

Suppose $i = k$. Then $h(e_k) = -1$.

For $v_k \in P_n$ we have

$$\sum_{e' \in N[e_k]} g(e') = g(e_{k-1}) + g(e_k) + g(e_{k+1}) + g(e_{k1}) + g(e_{km}) + g(e_{k+1\,1}) + g(e_{k+1\,m})$$

$$= 1 + (-1) + 1 + 1 + 1 + 1 + 1 = 5 \neq 1.$$

This is the case when a signed edge unidominating function fails an edge $e_k \in P_n$ where $h(e_k) = -1$ because it is in the vicinity of the edge.

Therefore h is not a signed edge unidominating function.

Since h is defined arbitrarily, there is no $h < g$ such that h is a signed edge unidominating function.

As a result, g is a minimal signed edge unidominating function.

g is the only minimal signed edge unidominating function because assigning the functional values $-1, 1$ to the edges of P_n and C_m in any other way does not make g any longer a signed edge unidominating function.

$$\text{Now } g(E) = \sum_{e' \in P_n} g(e') + \sum_{e' \in C_m} g(e')$$

$$= \underbrace{(1 + 1 + 1 \dots + 1)}_{(n-1)-times} + \underbrace{\underbrace{\{(1 + (-1) + 1) + (1 + (-1) + 1) + \cdots + (1 + (-1) + 1)\}}_{\left(\frac{m-2}{3}\right)-\text{times}} + 1 + 1}_{n-times}$$

$$= n - 1 + \left(\frac{m+4}{3}\right) n = \frac{1}{3}[mn + 7n - 3].$$

Thus $g(E) = \frac{1}{3}[mn + 7n - 3]$.

Now max $\{g(E)/g$ is a minimal signed edge unidominating function$\} = \frac{1}{3}[mn + 7n - 3]$, because g is the only one minimal signed edge unidominating function.

Therefore $\Gamma'_{su}(P_n \circ C_m) = \frac{1}{3}[mn + 7n - 3]$ when $m \equiv 2(mod\ 3)$.

Theorem 5.2: For $m \equiv 0(mod\ 3), m \equiv 1(mod\ 3), m \equiv 2(mod\ 3)$ the number of minimal signed edge unidominating functions of $P_n{}^\circ C_m$ is 1 with maximum weights

$\frac{1}{3}[mn + 3n - 3], \frac{1}{3}[mn + 5n - 3], \frac{1}{3}[mn + 7n - 3]$ respectively.

Proof: Follows by Theorem 5.1.

CONCLUSION:

In this paper the authors have studied signed edge unidominating function and minimal signed edge unidominating function of rooted product graph of a path with a cycle. This works throws light on further study of standard graphs and some other rooted product graphs.

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