Penetrative thermo-gravitational convection driven by surface tension in a ferrofluid layer subject to MFD viscosity via volumetric heat source

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Abstract - The effects of internal heat generation and magnetic field dependent (MFD) viscosity on the onset of Bénard–Marangoni ferroconvection (BMFC) subjected to the constant heat flux and constant temperature perturbations at the lower surfaces. The lower rigid surface is taken to be rigid either isothermal or insulating to temperature perturbations, upper surface is open to the atmosphere and subject to the general type of thermal boundary conditions. The eigenvalue problem is solved numerically by Galerkin technique (GT) and analytically by regular perturbation technique (RPT). It is noted that the combined effect of magnetic Rayleigh number and dimensionless internal heat source strength is to reinforce together and to hasten the onset of BMFC compared to their presence in isolation. The onset of FC is delayed with an increase in MFD viscosity parameter. In addition, nonlinearity of fluid magnetization is found to have no influence on the criterion for the onset of FC when both surfaces constant heat flux.

Keywords: Bénard–Marangoni convection, ferrofluids, heat transfer coefficient, Galerkin technique, magnetic field dependent viscosity, internal heat generation

I. INTRODUCTION

Ferrofluids (FFs) are commercially manufactured colloidal liquids usually formed by suspending mono-domain nano-particles (their diameter is typically 10 nm) of magnetite in non–conducting liquids like heptanes, kerosene, water, etc. and they are also called magnetic nanofluids. The FF is a type of functional fluid whose flow and energy transport processes may be controlled by adjusting an external magnetic field, which makes it find a variety of applications in various fields such as electronic packing, mechanical engineering, aerospace, bioengineering and thermal engineering. An authoritative introduction to this fascinating subject along with their applications is provided in Odenbach [1], Rosenwieg [2] and Shliomis [3].

The magnetization of FFs depends on the magnetic field, the temperature and the density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. This leads to convection in FFs in the presence of magnetic field gradient, known as FC, which is also known as Bénard-ferroconvection (BFC) (see Finlayson [4], Rosenwieg [5], Sekhar [6]). Convective instability in a FF layer can also be induced by surface tension forces provided it is a function of temperature and/or concentration. In view of the fact that heat transfer is greatly enhanced due to convection, the FC problems offer new possibilities for new applications in cooling of motors, loud speakers, transmission lines, and other equipment where magnetic field is already present. If the FF layer has an upper surface open to atmosphere then the instability is due to the combined effects of buoyancy as well as temperature–dependent surface tension forces, known as BMFC. Linear and nonlinear stability of combined buoyancy and surface-tension effects in a FF layer heated from below has been analyzed by Qin and Kalon [7]. The BMFC problems of FF layer heated from below under various assumptions is studied by many authors (Odenbach [8], Hennenberg et al. [9]-[11], Idris and Hashim [12], Nanjundappa et al. [13]-[16], Shivakumara et al. [17]). The effect of viscosity variations on the onset of BMFC in a horizontal layer of ferrofluid was investigated by Nanjundappa et al. [18], [19]. Recently, Sekhar et al. [20] have studied the effect of variable viscosity on thermal convection in Newtonian ferromagnetic liquid by considering different forms of boundary conditions. The effect of non-uniform basic temperature gradients on the onset of ferroconvection has been analyzed (Shivakumara and Nanjundappa [21], [22]). Nanjundappa et al. [23] have studied the effect of internal heat generation on the criterion for the onset of convection in a horizontal FF saturated porous layer Nanjundappa et al. [24] have explored a model for penetrative FC via internal heat generation in a FF saturated porous layer.

Marangoni convection (MC) arises when the surface tension force of an interface of fluid depends on the temperature. Schwab [25] experimentally examined the stability of plane layers of FFs for a vertical temperature gradient is applied. By using Galerkin method the resulting eigenvalue problem was solved. The authors discussed the mechanisms of suppressing and enhancing the FTC. It was found that the stability of BFMC was significantly affected by uniform temperature gradients. It was also found that the surface tension effect was negligible while the buoyancy was predominant force driving convection.

II. MATHEMATICAL FORMULATION

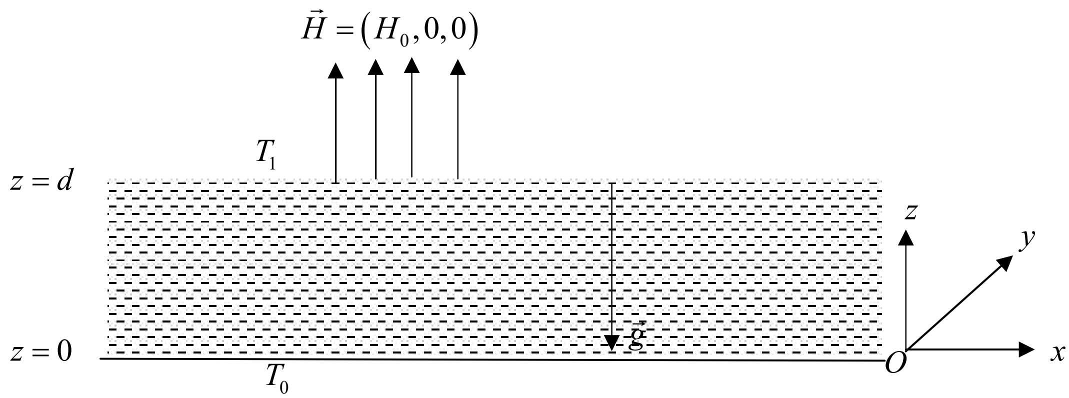

Figure1. Physical configuration

A system consisting layer of FF of width d in the existence of $\vec{H}=(H_0,0,0)$.A co-ordinate system OX, OY, OZ is chosen, OZ having perpendicularly upward directions and OX, OY in the horizontal plane. The gravity acts perpendicularly downward directions ($\vec{g}=-g\hat{k}$), where $\hat{k}$ is the unit vector in z-direction. The fluid is assumed to be incompressible having variable viscosity, given by $\eta=\eta_0(1+\vec{\delta}\cdot\vec{B})$, where $\vec{\delta}$ is the variation coefficient of magnetic field dependent viscosity and is considered to be isotropic, η_0 is taken as viscosity of the fluid when the applied magnetic field is absent and $\vec{B}=(B_x,B_y,B_z)$ is the magnetic induction. The constant temperatures of the fluid is considered to be confined between the surfaces, which are lower rigid for $T=T_0$ at $z=0$; upper free for $T=T_1(<T_0)$ at $z=d$. In addition, in energy equation, the internal heat generation (Q) effect cannot be abandoned however it is constant.

The surface tension, σ, and fluid density, ρ, are considered to linearly varying with temperature on the upper free surface,

$$\sigma=\sigma_0-\sigma_T\,(T-T_0). \qquad (1)$$

$$\rho = \rho_0\left[1-\alpha_t\left(T-T_0\right)\right] \quad (2)$$

The governing equations are

$$\nabla \cdot \vec{q} = 0 \quad (3)$$

$$\rho_0[\frac{\partial \vec{q}}{\partial t}+(\vec{q}\cdot\nabla)\,\vec{q}] = -\nabla \mathrm{p}+\rho_0\left[1-\alpha_t\,(T-T_0)\right]\vec{g}+ 2\nabla\cdot[\eta \underline{\underline{D}}]+\frac{\mu}{k}\vec{q}+\mu_0\,(\vec{M}\cdot\nabla)\vec{H} \quad (4)$$

$$\left[\rho_0 C_{V,H}-\mu_0\vec{H}\cdot\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt}+\mu_0 T\left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\cdot\frac{D\vec{H}}{Dt}=k_1\nabla^2 T+Q \quad (5)$$

$$\nabla\cdot\mu_0\left(\vec{M}+\vec{H}\right)=0, \nabla\times\vec{H}=0 \text{ or } \vec{H}=\nabla\varphi \quad (6)$$

$$\vec{M} = \frac{\vec{H}}{H}\left(M_0+\chi(H-H_0)-K(T-T_0)\right). \quad (7)$$

where $\vec{q}=\left(u,\,v,\,w\right)$ is the velocity, p the pressure, $C_{V,H}$ the specific heat at constant volume and magnetic field, $\vec{g}$ the acceleration due to gravity, $t$the time, ρ the density, $\rho_0$the mean density of the clean fluid, $\varepsilon$the porosity of the porous medium, $k$the permeability of the porous medium, α the coefficient of thermal expansion, $\mu_0$the free space magnetic permeability, $k_1$the thermal conductivity, $\underline{\underline{D}}=[\nabla\vec{q}+(\nabla\vec{q})^T]/2$the rate of strain tensor, $K=6\pi\tilde{\mu}\eta$, η being the particle radius, is the Stokes drag coefficient. The buoyancy effects are modeled by using the Boussinesq approximation. The last term on the right-hand side is the simplified expression of the Kelvin body force with convention $\vec{M}$ (ferrofluid magnetization) and $\vec{H}$ (magnetic field), H_0 the constant magnetic field, $H$the magnitude of $\left|\vec{H}\right|$, M the magnitude of $\vec{M}$, $M_0=M(H_0,\,T_0)$ the constant mean value of magnetization, ϕ the magnetic potential, m the mass of particles per unit volume, $\chi=(\partial M/\partial H)_{H_0,T_0}$the magnetic susceptibility, $Ta=(T_0+T_1)/2$ the average temperature, $\frac{d}{dt}\equiv\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}.\nabla\right)$ the convective derivative, the operator $\nabla\equiv\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ the Laplacian operator, the subscripts s and f represent the solid and fluid respectively.

In the basic state, we look for the solution in the form

$$\left[\vec{q}, \rho, p, T, \vec{H}, \vec{M}\right] = \left[0, \rho_b(z), \; p_b(z), T_b(z), \; \vec{H}_b(z), \vec{M}_b(z)\right] \quad (8)$$

where,

$$T_b(z) = -\frac{Q z^2}{2k_1} + \frac{Q d z}{2k_1} - \frac{\Delta T}{d} z + T_0 \quad (9)$$

$$\vec{H}_b(z) = \left[H_0 - \frac{K}{1+\chi}\left(\frac{Q z^2}{2k_1} - \frac{Q d z}{2k_1} + \frac{\Delta T}{d} z\right)\right]\hat{k} \quad (10)$$

$$\vec{M}_b(z) = \left[M_0 + \frac{K}{1+\chi}\left(\frac{Q z^2}{2k_1} - \frac{Q d z}{2k_1} + \frac{\Delta T}{d} z\right)\right]\hat{k}. \quad (11)$$

The distributed parabolically $T_b(z)$, $\vec{H}_b(z)$, $\vec{M}_b(z)$ on FF layer height due to the occurrence of internally heat source, Q . Nonetheless, the basic state distributions are linear in z for $Q = 0$ (see Fig. 2).

The basic state is perturbed once

$$(\vec{q}, \rho, p, T, \vec{H}, \vec{M}) = (0, \rho_b, p_b, T_b, \vec{H}_b, \vec{M}_b)(z) + (\vec{q}, \rho, p, T, \vec{H}, \vec{M})'(x, y, z, t) \quad (12)$$

Substituting into Eq. (6) yields

$$H_1 + M_1 = \left(1 + \frac{M_0}{H_0}\right) H_1$$

$$H_2 + M_2 = \left(1 + \frac{M_0}{H_0}\right) H_2$$

$$H_3 + M_3 = (1+\chi)\, H_3 - K\, T. \quad (13)$$

By substituting Eq. (12) into Eqs. (3)-(5), applying basic state Eq.(9)-(11) and ignoring the non-linear terms, we obtain

$$\left\{\rho_0 \frac{\partial}{\partial t} + \frac{\mu}{k} - \eta_b \nabla^2\right\}\nabla^2 w = -\mu_0 K \left\{Q\frac{z}{k_1} - Q\frac{d}{2k_1} + \beta\right\}\frac{\partial}{\partial t}\left(\nabla_h^2 \varphi\right) + \quad (14)$$
$$\rho_0 \alpha_t g \nabla_h^2 T + \frac{\mu_0 K^2}{1+\chi}\left\{-\frac{Q z}{k_1} + \frac{Q d}{2k_1} - \beta\right\}\nabla_h^2 T$$

$$\rho_0 C_0 \frac{\partial T}{\partial t} - \mu_0 T_0 K \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial z}\right) = k_1 \nabla^2 T + \left\{\rho_0 C_0 - \frac{\mu_0 T_0 K^2}{1+\chi}\right\}\left\{Q\frac{z}{k_1} - Q\frac{d}{2k_1} + \beta\right\} w \quad (15)$$

$$\left(1+\frac{M_0}{H_0}\right)\nabla_h^2\varphi+\left(1+\chi\right)\frac{\partial^2\varphi}{\partial z^2}-K\frac{\partial T}{\partial z}=0. \quad (16)$$

where $\eta_b=\eta_0[1+\Lambda\mu_0(M_0+H_0)]$ and $\nabla_h^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the horizontal Laplacian operator.

It is now assumed that the perturbation may be expressed in terms of their normal modes; thus

$$w=W(z)\exp\{ia_1x+ia_2x+\sigma t\} \quad (17a)$$

$$T=\Theta(z)\exp\{ia_1x+ia_2x+\sigma t\} \quad (17b)$$

$$\phi=\Phi(z)\exp\{ia_1x+ia_2x+\sigma t\} \quad (17c)$$

Non-dimensionaliing the resulting equations using the following scales are

$$\left(x*,\ y*,\ z*\right)=\left(\frac{x}{d},\ \frac{y}{d},\ \frac{z}{d}\right),\ W*=\frac{d}{\nu}W,\ \Theta*=\frac{\kappa}{\beta\nu d}\Theta,\ \Phi*=\frac{(1+\chi)\ \kappa}{\mathrm{K}\beta\nu d^2}\Phi,\ t*=\frac{\nu}{d^2}t,\ \sigma*=\frac{d^2}{\nu}\sigma \quad (18)$$

Substituting into Eqs. (14)-(16), then Eq.(17) is used to obtain the following equations are

$$(1+\Lambda)\left\{\left(D^2-a^2\right)^2-Da^{-1}\left(D^2-a^2\right)\right\}W-a^2R_m\{1-Ns(1-2z)\}\left(D\Phi-\Theta\right)-R_t\,a^2\,\Theta=0 \quad (19)$$

$$\left(D^2-a^2\right)\Theta+\{1-Ns(1-2z)\}W=0 \quad (20)$$

$$\left(D^2-M_3\,a^2\right)\Phi-D\Theta=0 \quad (21)$$

where $D=d/dz$ is the differential operator, $a=\left(a_1^2+a_2^2\right)^{1/2}$ horizontal wave number, $\Lambda=\ \delta\ \mu_0(M_0+H_0)$ is the non–dimensional magnetic field dependent viscosity parameter, $Da=k/d^2$ Darcy number, $M_1=\mu_0K^2\beta/(1+\chi)\rho_0\alpha g$ magnetic number, $R_t=\alpha\beta gd^4/\nu\kappa$ thermal Rayleigh number, $R_m(=R_tM_1)=\mu_0K^2\beta^2d^4/(1+\chi)\rho_0\nu\kappa$ magnetic Rayleigh number, $M_3=(1+M_0/H_0)/(1+\chi)$ non-linearity of fluid magnetization, $f(z)=Ns(1-2z)-1$ internal heat source strength. The classic value of M_2 is the order of 10^{-6} in different carrier liquids and hence its result is neglected.

It is considered that thelower boundary is rigid, while the upper free boundary at which the surface tension effects are accounted for is taken to be non-deformable and flat. In addition, both the boundaries are assumed to be perfectly insulated to temperature perturbations. The boundary conditions are then given by

(i) for conducting: $W = DW = 0,\ \Phi = 0,\ \ \Theta = 0$ at $z = 0$

(ii) for insulating: $W = DW = 0,\ \Phi = 0,\ D\Theta = 0$ at $z = 0$ (22)

(iii) for conducting/insulating :

$$W = (1+\Lambda)D^2W + Ma\ \ a^2\,\Theta = 0,\ D\Phi = 0,\ D\Theta + Bi\,\Theta = 0 \text{ at } z = 1 \tag{23}$$

III. METHOD OF SOLUTION

Equations (19)–(21) togetherwith (22) and (23) represent an eigenvalues R_m or R_t or Ma as aneigenvalue. Thus, the dependent variables are introduced the following base functions:

$$W = \sum_{i=1}^{N} A_i\,W_i(z),\ \ \Theta = \sum_{i=1}^{N} B_i\,\Theta_i(z),\ \ \Phi = \sum_{i=1}^{N} C_i\,\Phi(z) \tag{24}$$

Substituting into Eqs(19)–(21), then the multiplying on resulting equations respectively by $W_j(z), \Theta_j(z),\ \Phi_j(z)$ and on integrating, we get

$$F_{ji}A_i + G_{ji}B_i + H_{ji}C_i = 0 \tag{25}$$

$$I_{ji}A_i + J_{ji}B_i + K_{ji}C_i = 0 \tag{26}$$

$$L_{ji}A_i + M_{ji}B_i + N_{ji}C_i = 0 \tag{27}$$

where $F_{ji} - N_{ji}$ denotes the coefficients. That is

$$F_{ji} = (1+\Lambda)\left\{ < D^2W_j D^2W_i > + (2a^2 + Da^{-1}) < DW_j DW_i > + a^2(a^2 + Da^{-1}) < W_j W_i > \right\}$$

$$G_{ji} = -\frac{a^2}{4}Ma + a^2R_t < W_j\Theta_i > + a^2R_tM_1 < \{1 - Ns(1-2z)\} W_j\Theta_i >$$

$$H_{ji} = a^2R_tM_1 < \{1 - Ns(1-2z)\} W_j D\Phi_i >$$

$$I_{ji} = - < \{1 - Ns(1-2z)\}\,\Theta_j W_i >$$

$$J_{ji} = < D\Theta_j D\Theta_i + a^2\,\Theta_j\Theta_i > + \frac{Bi}{4}$$

$$K_{ji} = 0$$

$$L_{ji} = 0$$

$$M_{ji} = \langle \Phi_j D\Theta_i \rangle$$

$$N_{ji} = \langle D\Phi_j D\Phi_i \rangle + a^2 M_3 \langle \Phi_j \Phi_i \rangle$$

with $\langle \cdots\cdots \rangle = \int_0^1 (\cdots\cdots) dz.$

The linear system of Eqs. (25)-(27) can be reduced to

$$A\,X = 0, \tag{28}$$

where $A = \begin{bmatrix} F_{ji} & G_{ji} & H_{ji} \\ I_{ji} & J_{ji} & 0 \\ 0 & M_{ji} & N_{ji} \end{bmatrix}$ is the resulting matrix

and

$X = \begin{bmatrix} A_i \\ B_i \\ C_i \end{bmatrix}$ is the unknown column matrix.

Equation (28) can have non–trivial solutions if

$$\left| A \right| = 0. \tag{29}$$

The basis functions velocity W_i, magnetic potential Φ_i and temperature Θ_i will be chosen the following test functions corresponding to boundary conditions for extracting the eigenvalue from Eq.(29) :

$$W_i = (z^{i+3} - 5z^{i+2}/2 + 3z^{i+1}/2)\ ,\ \Phi_i = (z - z^{i+1}/2)$$

$$\Theta_i = \begin{cases} z^i - z^{i+1}/2 & \text{for conducting case} \\ z^{i-1} & \text{for insulating case} \end{cases} \tag{30}$$

The above test functions satisfy the conditions (22) and (23) at $z = 0,\ 1$ but the residue is included from the differential equations. On substituting we get

$$F\left\{R_t,\ R_m,\ Ma,\ Da^{-1},\ M_1, M_3,\ \Lambda,\ Ns,\ a\right\} = 0 \tag{31}$$

provides relationships between the parameters $M_1,\ Da^{-1},\ M_3,\ \Lambda,\ Ns$ and thus determines the minimum of R_t or Ma or R_m by corresponding a_c.

IV. RESULTS AND DISCUSSIONS

A linear stability analysis is carried out to investigate the combined effect of internal heat generation and MFD viscosity on the onset of coupled Bénard-Marangoni convection in a horizontal ferrofluid layer in the presence of a uniform vertical magnetic field. The values (R_{tc}, a_c) obtained from the current study under the limiting cases are compared with Sparrow et.al. [26] in Table 1 for selected Bi with $M_1 = \Lambda = Ns = 0$ and $M_3 = 1$ and Char and Chiang [27]for different Ns with $M_1 = \Lambda = 0, M_3 = 1$ and $Bi\ (= 0, \infty)$ in Table 2. From these Tables 1 and 2, it is clear that there is an admirable concurrence between the current and earlier published ones.

Table-1 Comparison of $(R_{tc},\ a_c)$ for conducting and insulating case $M_1 = \Lambda = Ns = 0$ and $M_3 = 1$

	Conducting Case				**Insulating Case**			
	Present study		**Sparrow et.al. [26]**		**Present study**		**Sparrow et.al [26]**	
Bi	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c
0	668.998	2.085	669.001	2.09	320	0	320.00	0
0.1	682.360	2.116	682.361	2.115	381.665	1.015	381.665	1.015
0.3	706.368	2.168	706.365	2.17	428.290	1.299	428.290	1.30
1	770.569	2.292	770.569	2.30	513.789	1.643	513.792	1.64
3	872.502	2.451	872.506	2.46	619.666	1.921	619.666	1.92
10	989.491	2.588	989.493	2.59	725.147	2.105	725.150	2.11
30	1055.347	2.648	1055.345	2.65	780.237	2.176	780.240	2.18
100	1085.897	2.671	1085.893	2.67	804.971	2.202	804.973	2.20
∞	1100.649	2.682	1100.657	2.68	816.744	2.214	816.748	2.21

Table-2 Comparison of $\left(R_{tc},\ a_c\right)$ for conducting and insulating case $M_1 = \Lambda = 0$ and $M_3 = 1$.

	$Bi = 0$ **Conducting**				$Bi = \infty$ **Insulating**			
	Present study		**Char and Chiang [27]**		**Present study**		**Char and Chiang [27]**	
Ns	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c
0	668.998	2.085	668.998	2.086	668.998	2.085	668.998	2.086
0.5	608.745	2.070	608.746	2.070	608.745	2.070	608.746	2.070
1	557.606	2.059	557.607	2.060	557.606	2.059	557.607	2.060
5	328.582	2.034	328.582	2.035	328.582	2.034	328.582	2.035
10	215.409	2.032	215.409	2.035	215.409	2.032	215.409	2.035
15	159.952	2.034	159.952	2.034	159.952	2.034	159.952	2.034
20	127.140	2.035	127.14	2.036	127.140	2.035	127.14	2.036
30	90.113	2.037	90.114	2.038	90.113	2.037	90.114	2.038
40	69.776	2.039	69.776	2.039	69.776	2.039	69.776	2.039
70	41.569	2.041	41.597	2.042	41.569	2.041	41.597	2.042
100	29.628	2.042	29.628	2.043	29.628	2.042	29.628	2.043

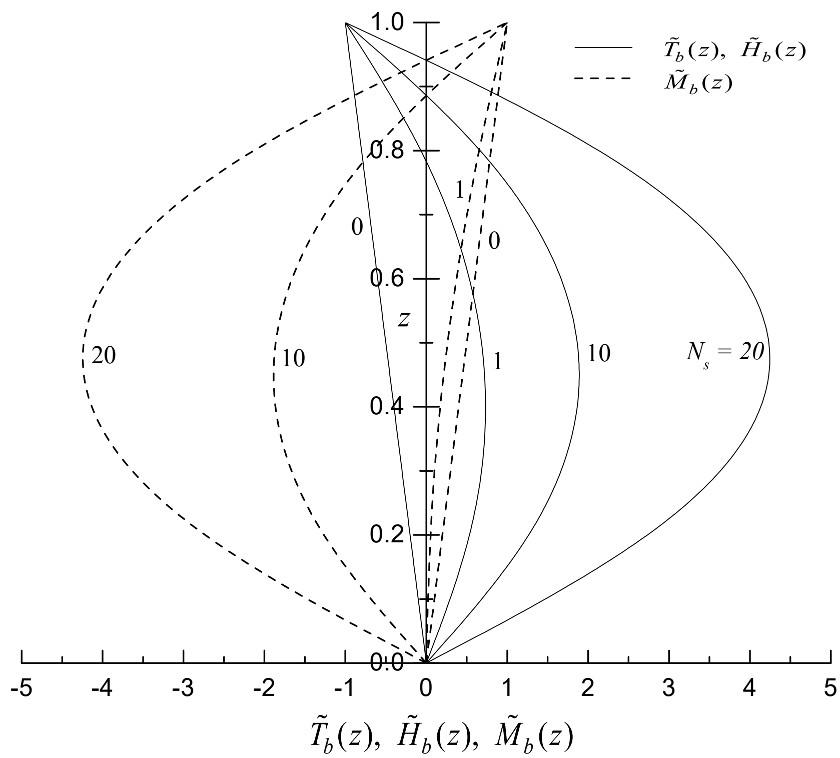

Figure 2. Basic state distributions

Welook into the simultaneously arising instabilities had been caused by three thermo-physical mechanisms: (i) gravitational buoyancy, R_{tc}, (ii) magnetic pondermotive, R_m, and (iii) surface tension, Ma, forces acting on the system. In Figs. 3-13 gives an illustration representation of the linear stability thresholds for solid lines represent the pure Bénard-convection ($Ma = 0$) and dashed lines represent the pure Marangoni convection ($R_t = 0$). In Fig. 3, plot of critical values (R_{tc}, Ma_c) against Da^{-1} for $M_1 = Ns = Bi = 2$ and $M_3 = \Lambda = 1$. As one would physically, R_{tc} is more stable than Ma_c as the permeability is increasing. That is $Ma_c < R_{tc}$. The effect of increasing Da^{-1} is to delay the onset ofBM convection.

In Fig. 4 shows that increase R_{tc} with increase in MFD viscosity, Λ, since viscous magnetic increase in number with Λ greater part of the energy of the system is consumed by this number and as a result the onset of BMC is delayed.

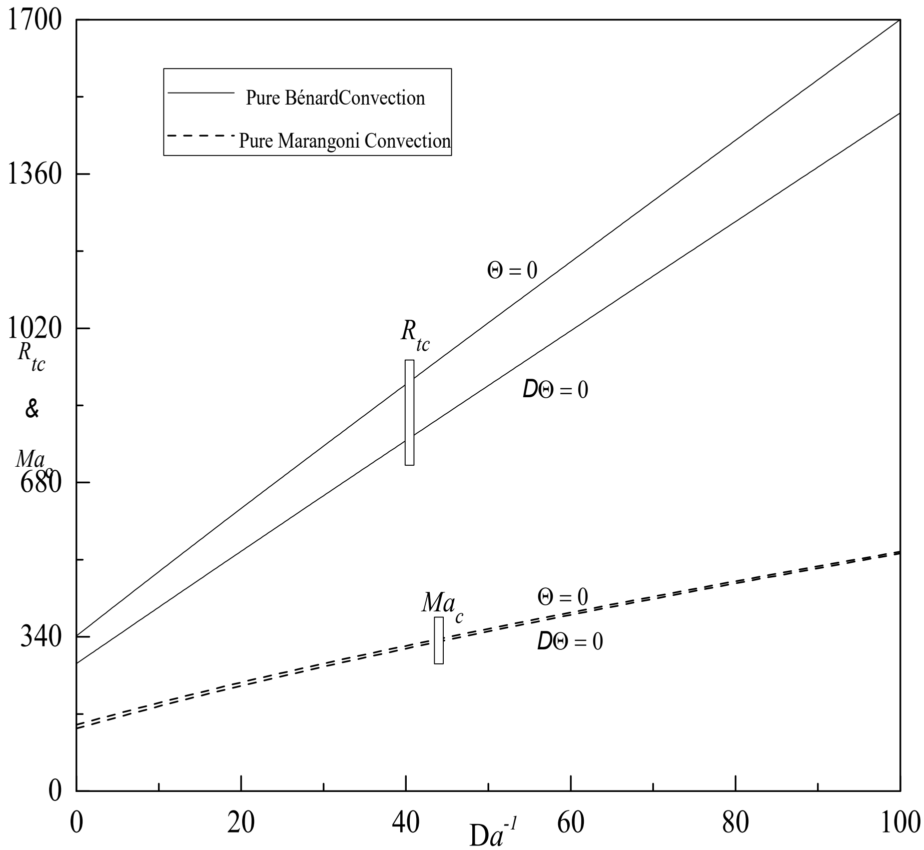

Figure 3. Variation of R_{tc} and M_{ac} versus Da^{-1} for $Ns = M_1 = Bi = 2$, $M_3 = 1$ and $\Lambda = 1$

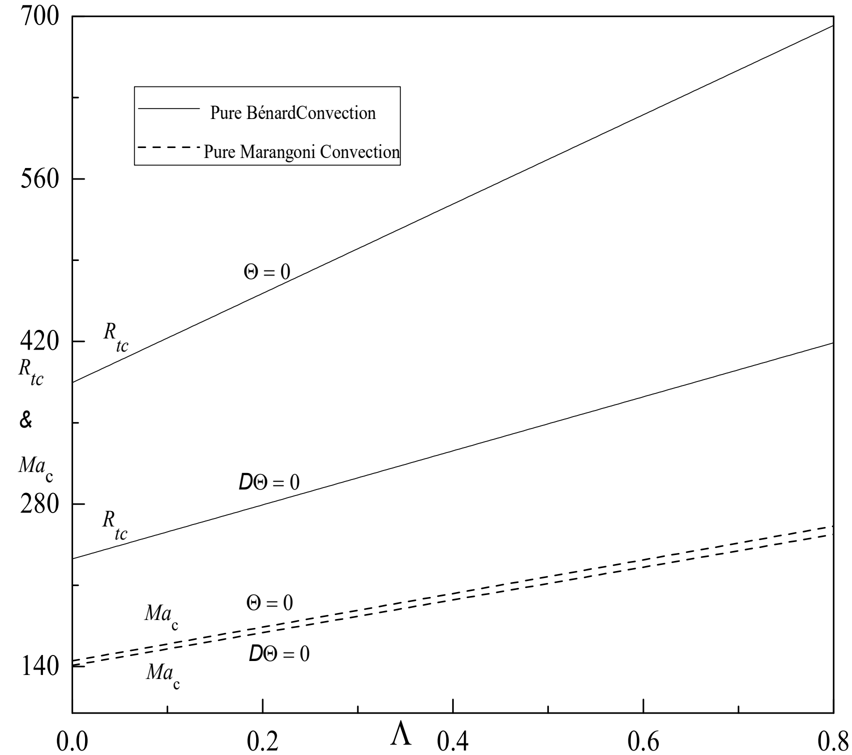

Figure 4. Variation of R_{tc} and M_{ac} versus Λ for $Ns = M_1 = Bi = 2,\ M_3 = 1$ and $Da^{-1} = 25$

In Fig. 5, M_3 stands the departure of magnetic equation of state from linearity. As the equation of state turns into more non-linear (M_3 large), the fluid layer is destabilized slightly. However, in the case of upper insulating ($D\Theta = 0$), effect of M_3 have no influence on the measure for the onset of FTC.

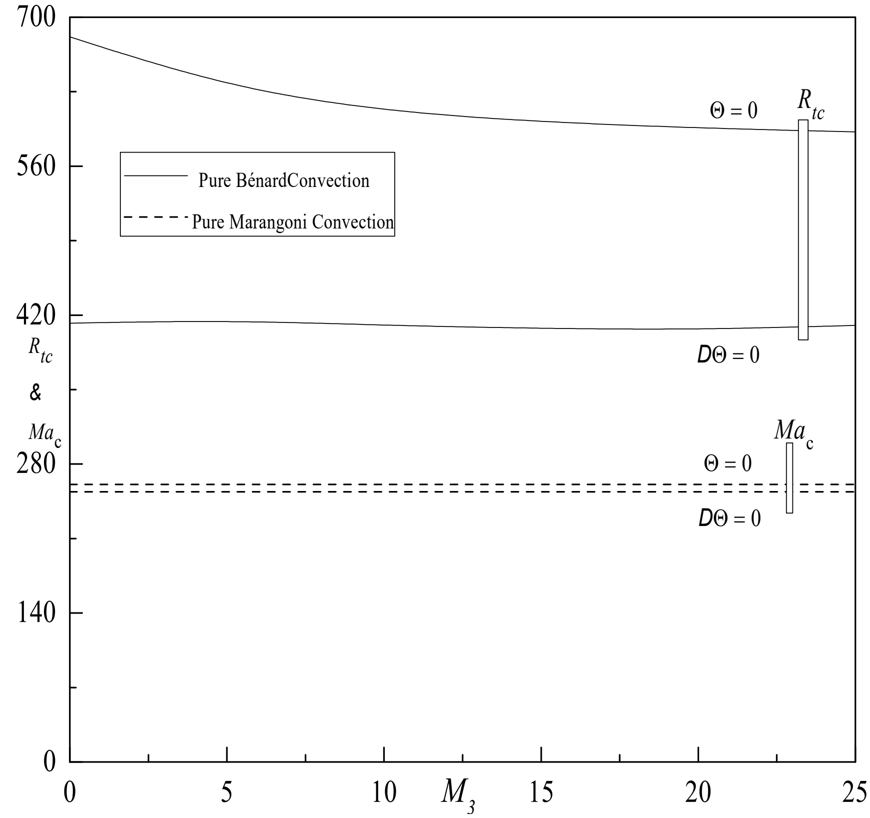

Figure 5. Variation of R_{tc} and M_{ac} versus M_3 for $Ns = M_1 = Bi = 2,\ \Lambda = 1$ and $Da^{-1} = 25$

The graphs of R_{tc} and Ma_c against internal heat source Ns for $M_1 = Bi = 2,\ \Lambda = M_3 = 1$ and $Da^{-1} = 25$ is depicted in Fig. 6. As illustrated that the impact of increasing Ns parameter suppressed the R_{tc} and Ma_c monotonically and destabilize the system. From the Fig. 7 it is clear that an increase in Bi, R_{tc} and Ma_c increases, thus its effect is to delay the onset. This may perhaps be attributed to fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a enhanced convective heat transfer coefficient, h_t, at the upper surface and hence elevated heating is necessary to make the system unstable.

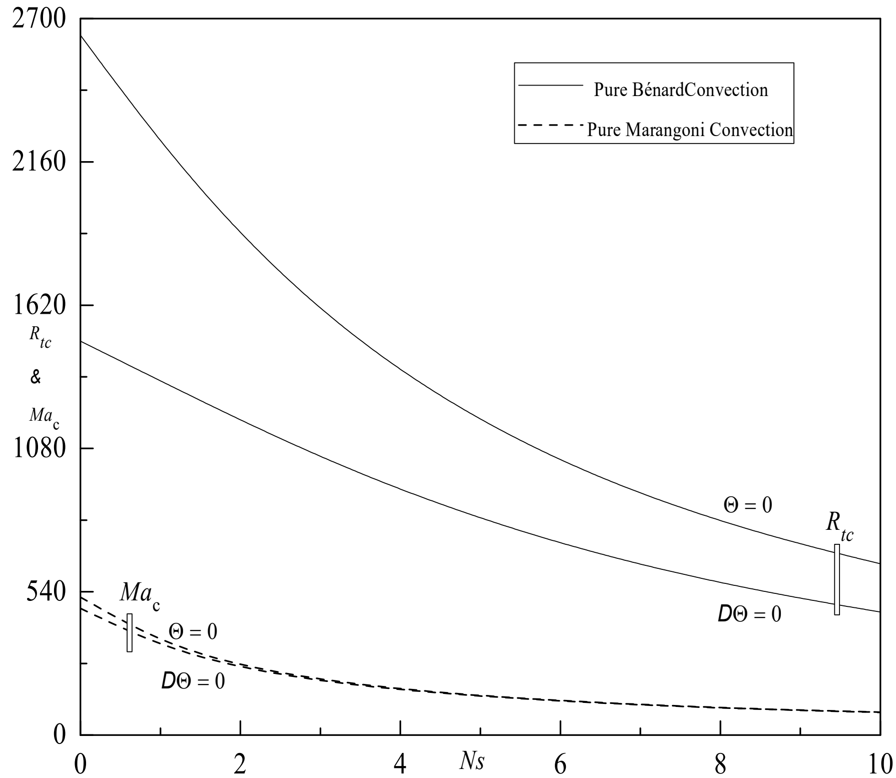

Figure 6. Variation of R_{tc} and M_{ac} versus Ns for $M_3 = \Lambda = 1$, $M_1 = Bi = 2$ and $Da^{-1} = 25$

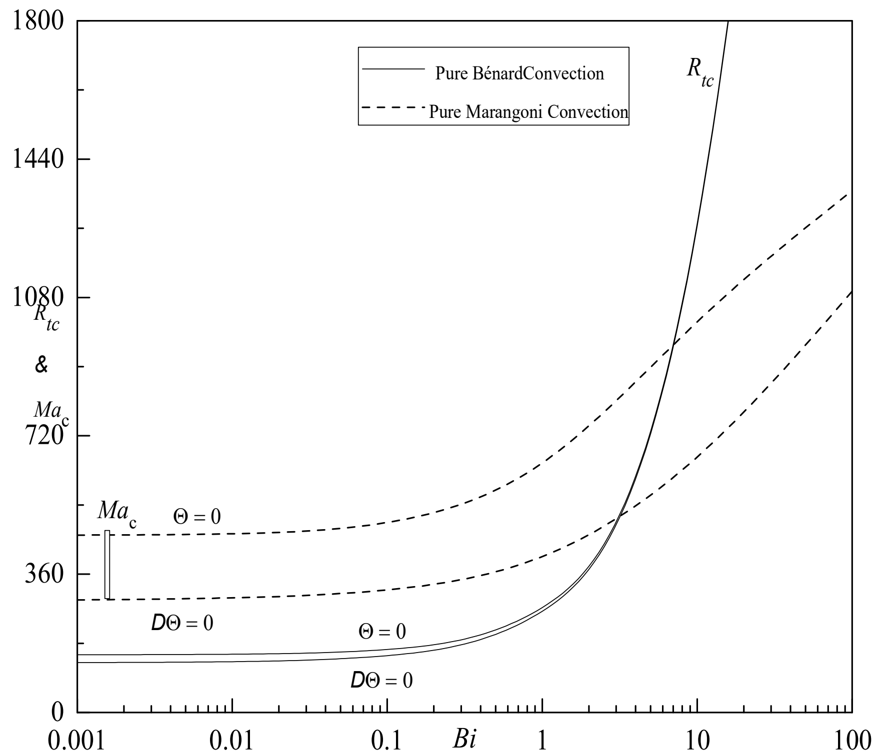

Figure 7. Variation of R_{tc} and M_{ac} versus Bi for $M_3 = \Lambda = 1$, $M_1 = Ns = 2$ and $Da^{-1} = 25$

The effect of the magnetic parameter, M_1 ,on R_{tc} and Ma_c are shown Fig. 8. From the graph, the stability in a horizontal FF layer on the onset of BMC is significantly altered by effect. R_{tc} and Ma_c decreases with increasing M_1 for $Ns = Bi = 2,\ M_3 = \Lambda = 1$ and $Da^{-1} = 25$, and thus it hastens the onset of BMC due to increasing in magnetic force. It is importance reveals that the value of R_{tc} recorded for isothermal surface, $\Theta = 0$, is the highest and this indicates that the isothermal boundaries is the most stable compared to the insulating surfaces, $D\Theta = 0$.

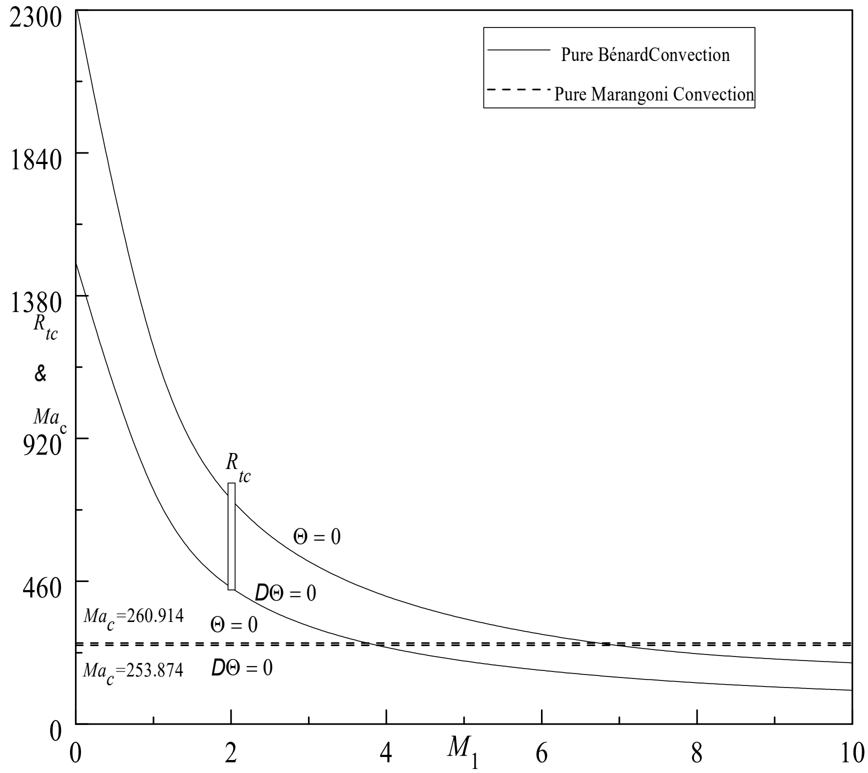

Figure 8. Variation of R_{tc} and M_{ac} versus M_1 for $Ns = Bi = 2,\ M_3 = \Lambda = 1$ and $Da^{-1} = 25$

Figure 9 shows the relation between Da^{-1} and a_c, it pointed to that increase in Da^{-1} have a tendencies to increase a_c, thus effect is to lessen size the convection cell. A plot of a_c varies with Λ (Fig. 10), it is clearly seen that a_c has no changes for pure Marangoni case and is to delay the onset.

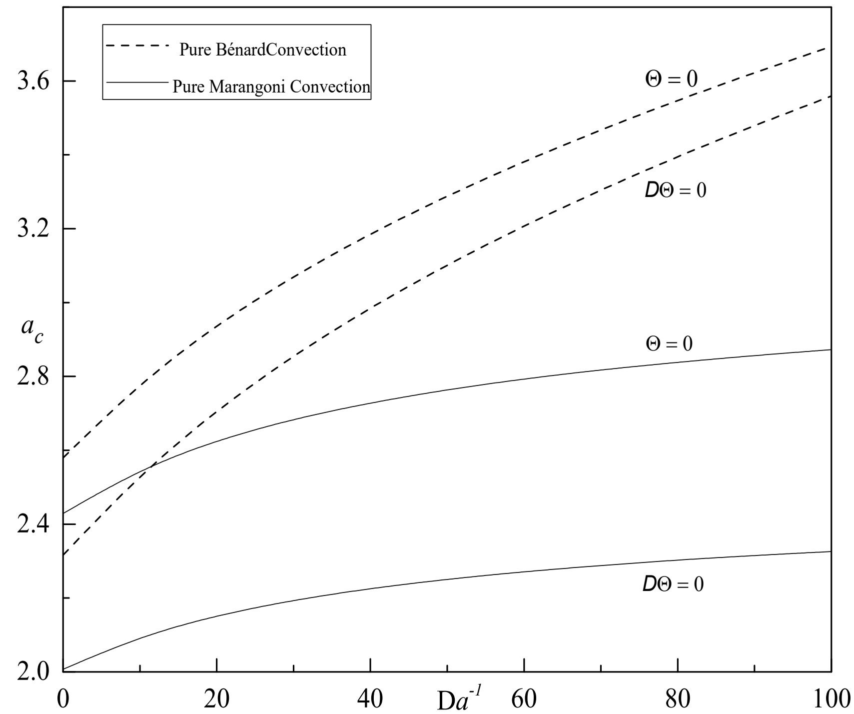

Figure 9. Variation of a_c against Da^{-1} for $M_1 = Ns = Bi = 2$ and $M_3 = \Lambda = 1$

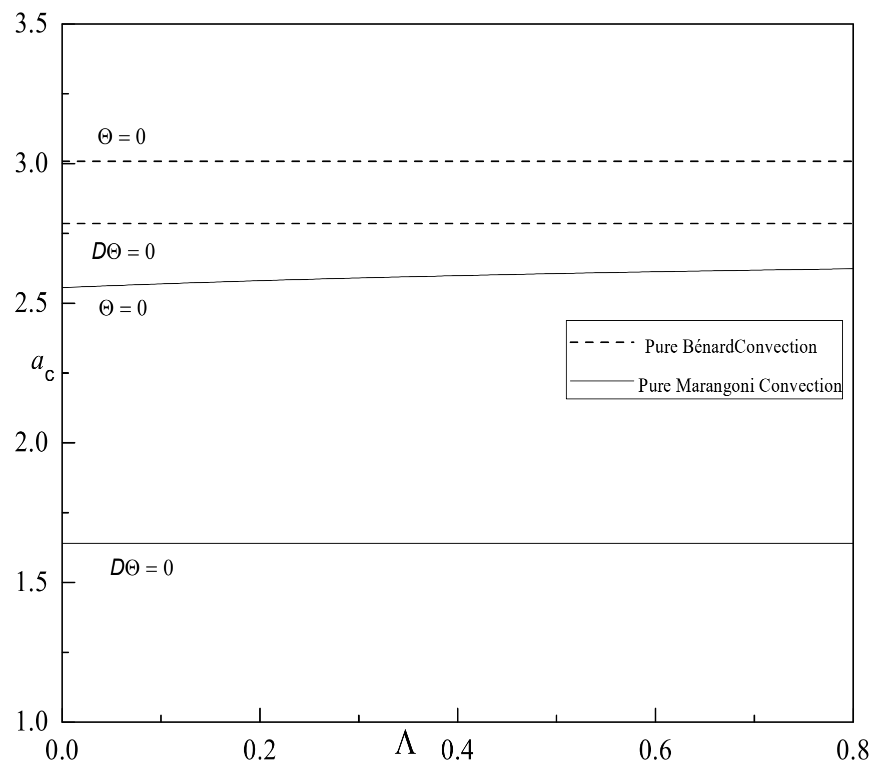

Figure 10. Variation of a_c against Λ when $M_3 = 1$, $M_1 = Bi = Ns = 2$ and $Da^{-1} = 25$

Figure 11 shows that the relative between M_1 and a_c, it designated that increase in M_1 tends to increase a_c, thus effect is to lessen the convection cell size. However, a_c have no alter for pure Marangoni case. In Fig. 12 as a_c decreases with increase in M_3 this imply convection cell size become large for pure Bénard case with increasing M_3, however a_c have no change for pure Marangoni case. Thus a_c does not depend on M_3.

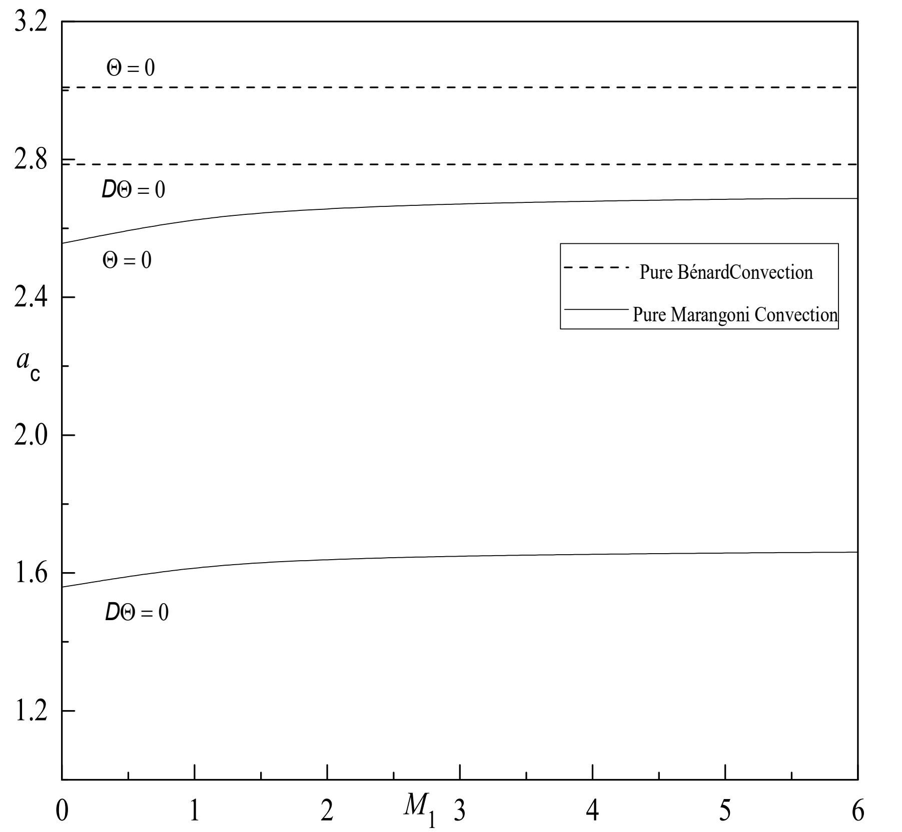

Figure11. Variation of a_c against M_1 when $\Lambda = M_3 = 1,\ Ns = Bi = 2$ and $Da^{-1} = 25$

Figure 13 shows the relation between Ns and a_c, it indicates that increase in Ns tends to increase a_c, thus reduce the convection cell size. In fig. 14 increase in Bi is to decrease the width of convection cells. The a_c values for Bénard FTC are constantly found to be higher than Marangoni FTC.

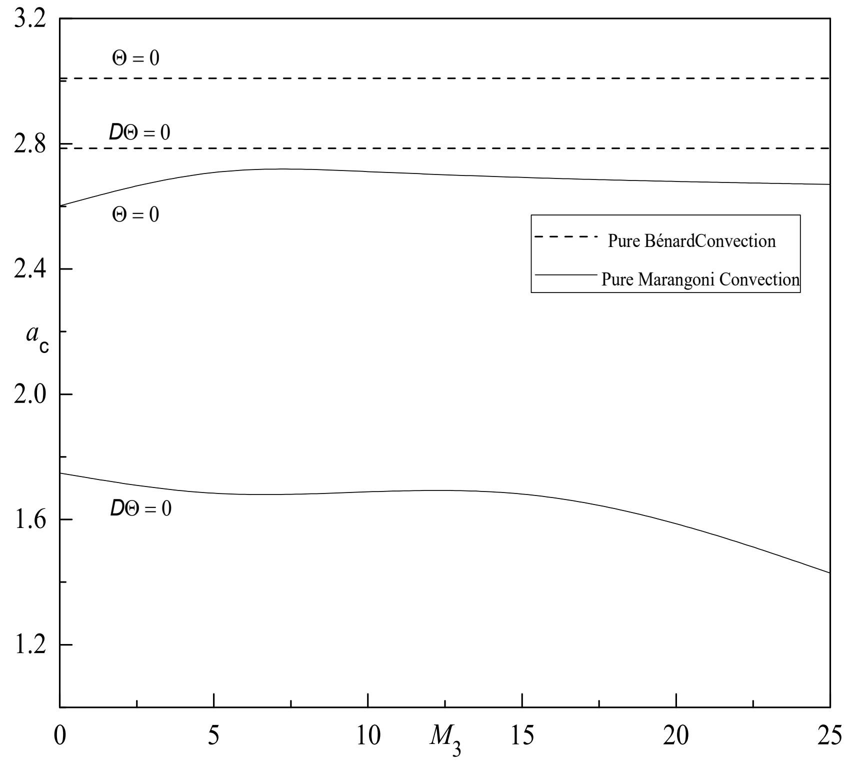

Figure 12. Variation of a_c against M_3 for $\Lambda = 1,\ M_1 = Bi = Ns = 2$ and $Da^{-1} = 25$

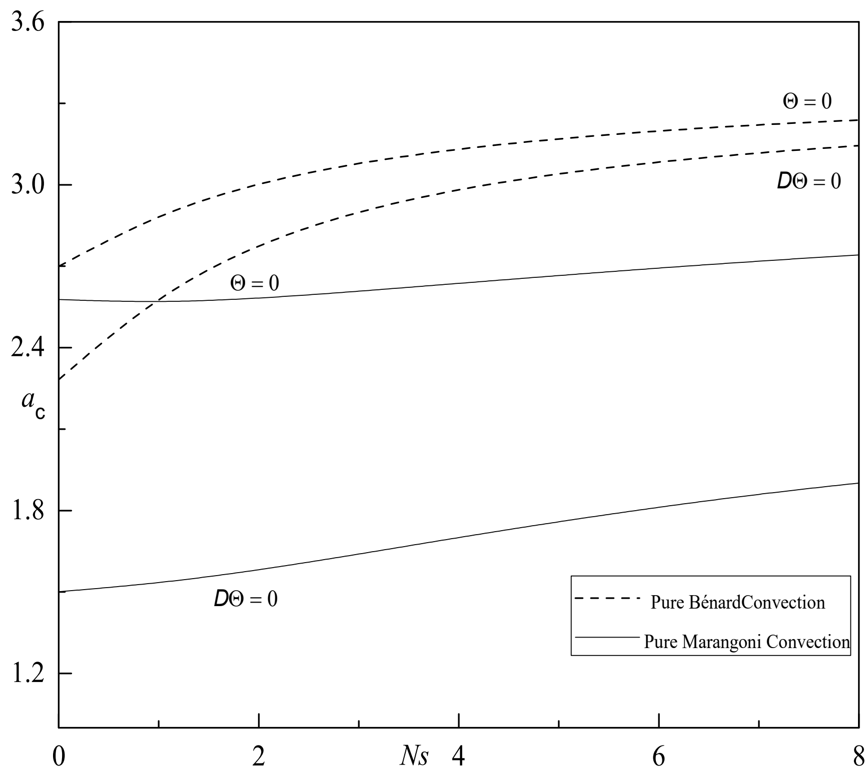

Figure 13. Variation of a_c against Ns for $\Lambda = M_3 = 1,\ M_1 = Bi = 2$ and $Da^{-1} = 25$

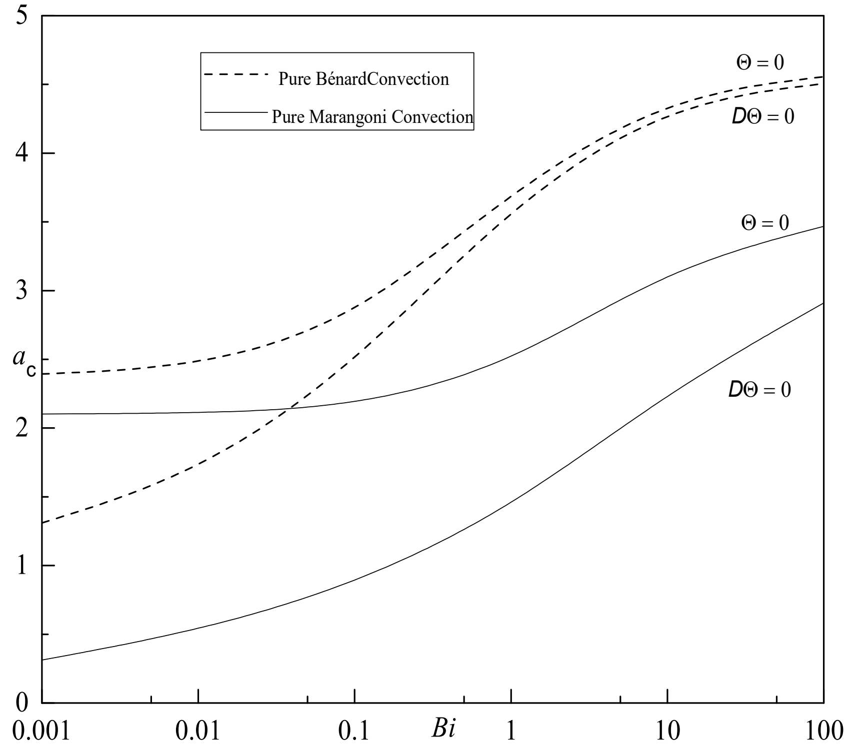

Figure 14. Variation of a_c against Bi for $\Lambda = M_3 = 1,\ M_1 = Ns = 2$ and $Da^{-1} = 25$

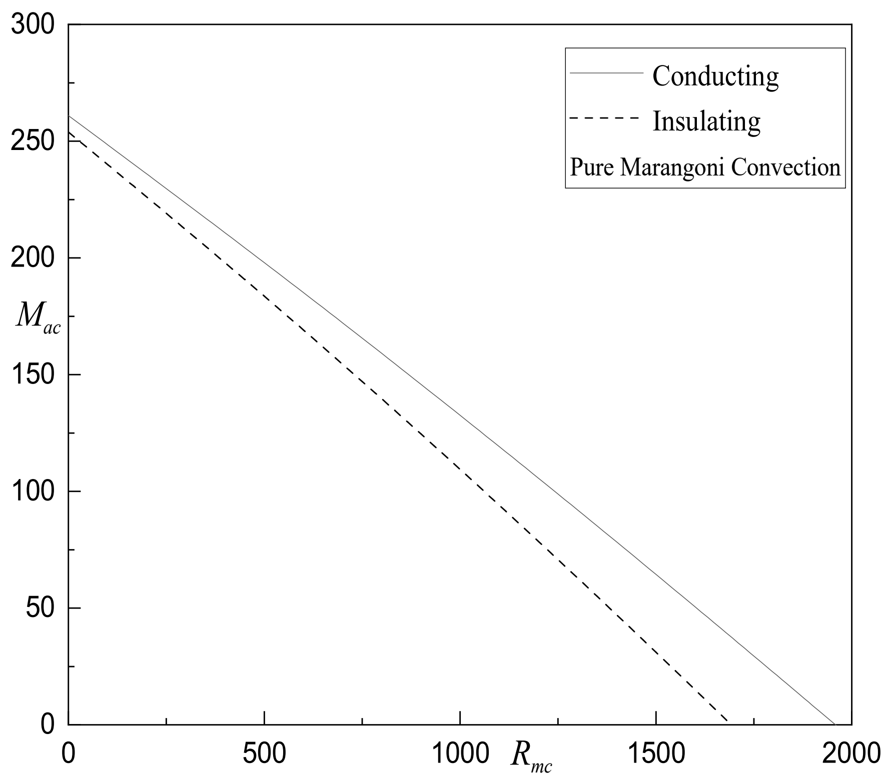

Figure 15(a). Locus of Ma_c against R_{mc} for $\Lambda = M_3 = 1,\ M_1 = Bi = 2,\ \ Da^{-1} = 25$ and $Ns = 2$

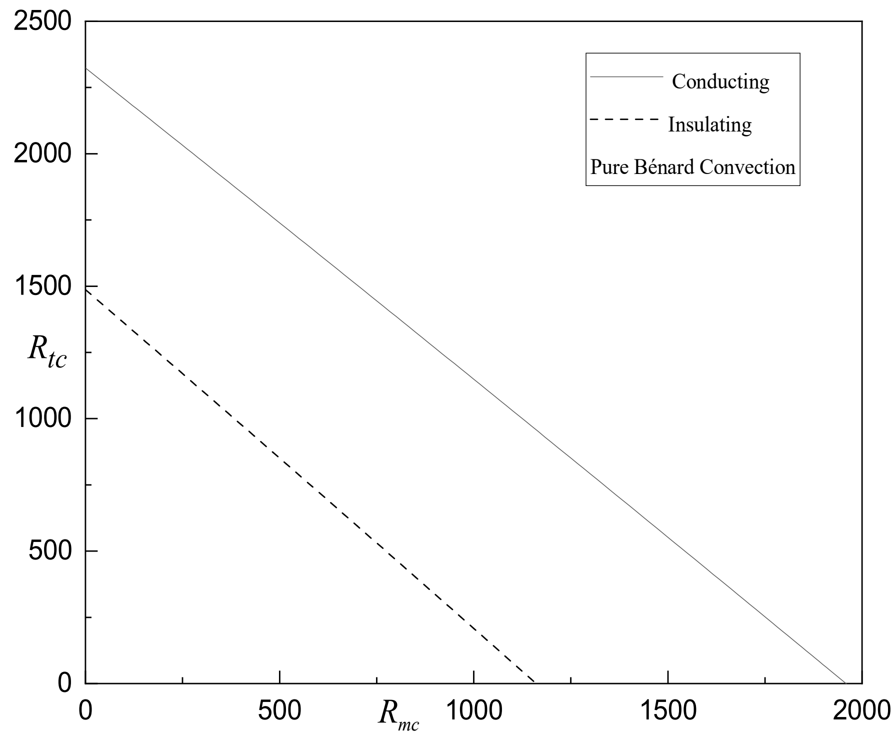

Figure 15(b). Locus of R_{tc} against R_{mc} for $\Lambda = M_3 = 1$, $Bi = M_1 = Ns = 2$ and $Da^{-1} = 25$

Figures shows the locus 15(a) Ma_c and R_{mc}, and 15(b) R_{tc} and R_{mc} for different thermal bounding surfaces. It is evident there is a strong coupling between R_{tc} or R_{mc} or Ma_c an increase in R_{mc} has a destabilizing effect on the onset.

V. CONCLUSIONS

The onset of penetrative BMFC in a FF layer is investigated theoretically via internal heating with MFD viscosity. The lower rigid and the upper horizontal free boundaries are considered to be perfectly insulated to temperature perturbations. The combined effect of internal heat source strength measured through the parameter Ns and MFD viscosity parameter Λ on the stability characteristics of the system is analyzed in detail. The effect of increase in MFD viscosity parameter Λ is to delay the onset of BMFC, while increase in magnetic Rayleigh number R_m and dimensionless internal heat source strength Ns is to reinforce together and to hasten the onset. Thus MFD viscosity plays a crucial role in controlling BMFC. The nonlinearity of fluid magnetization parameter M_3 has no effect on the onset. The buoyancy and surface tension forces complement with each other and it is always found that $Ma_c < R_{tc}$; a result in accordance with ordinary viscousfluids.

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