COMPUTATIONAL APPROACH FOR TRANSIENT BEHAVIOUR OF FINITE SOURCE RETRIAL QUEUEING MODEL WITH IMPATIENT CUSTOMERS

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Abstract- we are analyzing a finite source retrial queueing system with impatient occupants. All transitions are assembled by using an infinitesimal generator matrix. Eigen values and Eigen vectors are used to obtain the Time dependent and Steady state solutions. Computational studies have been done for analysis of Time dependent mean number of occupants in the orbit, Transient probabilities of server free/busy for various values of $M, \lambda, \mu, \sigma, \delta$ and t .

Keywords – Finite source queues – Reneging customers – Retrial queue – Classical Retrial policy – Infinitesimal matrix – Eigen values – Eigen vectors – Exponential of a matrix – Time dependent solutions.

I. INTRODUCTION

The main objective of this model is to analyze the transient behaviour of finite source Retrial queueing system by new computational approach. Incoming occupants who find all the servers are busy then after some random time they retry for service which is known as **Retrial queues**. Studies on Retrial queues by Falin and Templeton (1997). Retrial queues and its studies have been found in Templeton and Yang (1987), Falin (1990), Artalejo (2010).

Studies on a Finite source queues have received considerable attention from researchers such as Patrick Wüchner, János Sztrik, , Hermannde Meer (2009). Some of the works for transient behavior studies done by Sophia (2008) have analysed the Transient analysis for state dependent queues with catastrophes and further in 2016, she analyzed Transient Analysis of a Discouraged Arrival with some parameters.

Rakesh Kumar, Sapana Sharma (2018) has analysed Transient performance analysis of a Single Server Reneging customers, Parthasarathy and Sudhesh (2008) have analysed the Transient solution of a multi server with *N*-policy. Sudhesh (2010) have analysed the Transient analysis of a queue with system disasters and impatience. Sherif Ammar (2015) analysed the Transient analysis of an M/M/1 queue with impatient behaviour and multiple vacation and further in 2017 he analysed Transient solution of M/M/1 vacation queue. Computational Approach of an infinitesimal generator matrix study was done by Neuts (1981) has analysed Matrix Geometric Solutions in Stochastic Models.

II. THE MATHEMATICAL MODEL AND ITS SOLUTIONS

2.1Model Description **-**

Consider a retrial queueing system with Reneging customers of finite source in which the primary arrival rate λ which follows a Poisson distribution and the service time μ follows an exponential distribution. Further, we have assumed that the calling population is finite of size M . If the server is idle then the primary arrival will be served immediately and after completion of the service, it leaves the system. If the server is not idle then the arriving occupant goes to orbit becomes a group of repeated occupants. This group of source of repeated occupants may be viewed as a sort of queue. Every such source produces a Poisson process of repeated occupants with intensity σ. If an incoming repeated arrivals find that the server idle, then they served and leaves the system after service, therefore this repeated occupants disappears. However, an occupant from the orbit may leave the orbit due to impatience after some random time for not getting service even after continuously retrying; this process is called the impatient customers with parameter d follows an exponential distribution.

2.2 Retrial Policy-

The retrial policy states that the probability of an occupants from an orbit to try for service during the time interval $(t, t+\Delta t)$ given that there were n occupants in orbit at time t is $n\sigma\,\Delta t + O(\Delta t)$. This discipline for access for the server from the retrial group is called classical retrial rate policy**.**

2.3 Representation of Random Process-

Define:

1. N (t) – which denotes customers coming into the system at time t and
2. C (t) - which denotes if the server is free/idle at time t.

The random process is described as {< N(t) ,C(t)> / N(t) = 0,1,2,3,…,M-1; C(t) = 0,1}, where

C (t) = 0 if the server is idle at time t & C (t) = 1 if the server is busy at time t

Kolmogorov balanced equations are given below

$$\left.\begin{aligned} p'_{00}(t) &= -M\lambda p_{00}(t) + \mu p_{01}(t) \\ p'_{01}(t) &= -((M-1)\lambda + \mu) p_{01}(t) + M\lambda p_{00}(t) + \sigma p_{10}(t) + \delta\, p_{11}(t) \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} p'_{n0}(t) &= -((M-n)\lambda + n\sigma) p_{n0}(t) + \mu p_{n1}(t) \\ p'_{n1}(t) &= -((M-n-1)\lambda + \mu + n\delta) p_{n1}(t) + (M-n)\lambda p_{n0}(t) + (M-n)\lambda p_{n-11}(t) \\ &\quad + (n+1)\sigma p_{n+10}(t) + (n+1)\delta p_{n+11}(t) \\ &\qquad \text{where } n = 1,2,3,\ldots,M-2 \end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned} p'_{M-10}(t) &= -((M-(M-1)\lambda + (M-1)\sigma) p_{M-10}(t) + \mu p_{M-11}(t) \\ p'_{M-11}(t) &= -[(M-M)\lambda + \mu + (M-1)\delta] p_{M-11}(t) + [M-(M-1)]\lambda p_{M-10}(t) + [M-(M-1)]\lambda p_{M-21}(t) \end{aligned}\right\} \quad (3)$$

Continuous Markov chain process with state space is given by *{(u, v) / u = 0, 1, 2, 3, . . ., M-1; v = 0, 1}*

The infinitesimal generator matrix (IGM) *MAT* for this model is given below

$$MAT = \begin{pmatrix} L_{00} & L_{01} & 0 & 0 & 0 & \ldots\ldots & L_{0M\text{-}1} \\ L_{10} & L_{11} & L_{12} & 0 & 0 & \ldots\ldots & L_{1M\text{-}1} \\ 0 & L_{21} & L_{22} & L_{23} & 0 & \ldots\ldots & L_{2M\text{-}1} \\ 0 & 0 & L_{32} & L_{33} & L_{34} & \ldots\ldots & L_{3M\text{-}1} \\ . & . & . & . & . & \ldots\ldots & \ldots\ldots.. \\ L_{M\text{-}10} & L_{M\text{-}11} & L_{M\text{-}12} & L_{M\text{-}13} & L_{M\text{-}14} & \ldots\ldots. & L_{M\text{-}1M\text{-}1} \end{pmatrix}$$

The matrices $L_{00,}L_{01}, L_{10}, L_{11}, L_{21}, L_{22}, L_{32}, L_{33}, \ldots, L_{M\text{-}1M\text{-}1}$ are described in the IGM *MAT*. The infinitesimal transition rates of process ***X*** as follows

$$L_{i\,i} = \begin{bmatrix} -((M-i)\lambda + i\sigma) & \mu \\ (M-i)\lambda & -(M-(i+1))\lambda + \mu \end{bmatrix} \quad for\ i = 0,1,2,\ldots,M-1$$

$$L_{i\,i+1} = \begin{bmatrix} 0 & 0 \\ (i+1)\sigma & (i+1)\delta \end{bmatrix} \quad for\ i = 0,1,2,\ldots,M-2$$

$$L_{i\,i\text{-}1} = \begin{bmatrix} 0 & 0 \\ 0 & (M-i)\lambda \end{bmatrix} \quad for\ i = 1,2,\ldots,M-1$$

The equations (1) – (3) can be combined and expressed as

$$X'(t) = LX(t),$$

$$where\ L = MAT^T\ \&\ [X(t)]^T = [P_{00}(t)\quad P_{01}(t)\quad P_{10}(t)\quad P_{11}(t)\quad .\ .\ .\ .\ .P_{M\text{-}1\,0}(t)\ \ P_{M\text{-}11}(t)]$$

Solving the equations, we get, $X(t) = e^{Lt}\,X_0$

When t = 0, $X_0 = X(0) = [1\quad 0\quad 0\quad 0\quad 0\quad .\ .\ .\quad 0\quad 0\quad 0]^T$

We define,

$P_{n0}(t)$: Probability that the server is idle for n no. of arrivals at time t in the orbit.

$P_{n1}(t)$: Probability that the server is busy for n no. of arrivals at time t in the orbit.

2.4 Description of Computational Method-

The following effective computational procedure is used to find the Time dependent probabilities of number of customers in the orbit at time *t*. The Time dependent Probabilities is denoted by

$$X(t) = [P_{00}(t), P_{01}(t), P_{10}(t), P_{11}(t), P_{20}(t), P_{21}(t), ..., P_{M-10}(t), P_{M-11}(t)]^T$$

Step 1: Find the Eigen values and Eigen vectors of this finite order matrix *tL*.

Step 2: Let $d_1, d_2, d_3, ..., d_{2M}$ be *2M* Eigen values and $\overline{c_1}, \overline{c_2}, \overline{c_3}, ..., \overline{c_{2M}}$ be *2M* Eigen vectors.

Step 3: Represent this Eigen vectors as column vectors of a matrix $C = \left(\overline{c_1}, \overline{c_2}, \overline{c_3}, ..., \overline{c_{2M}}\right)$.

Step 4: Let

$$D = \begin{pmatrix} d_1 & 0 & 0 & . & . & 0 \\ 0 & d_2 & 0 & . & . & 0 \\ 0 & 0 & d_3 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & d_{2M} \end{pmatrix}$$

Step 5: Find the Exponential of the matrix *tL* using *D* and *C*.

Step 6: Extract the first column of this Exponential matrix *tL* and store in *X (t)*.

Step 7**:** This probability vector in *X (t)* provides time dependent probabilities of number of customers in the orbit at time *t*.

2.5 System performance measures

The following system measures are used to bring out the Transient behaviour of the retrial queueing model under Computational study to the following measures for various values of $M, \lambda, \mu, \sigma, \delta$ and t.

a. Probability that the server is idle at time t = $P_{idle}(t) = \sum_{n=0}^{\infty} P_{n0}(t)$

b. Probability that the server is busy at time t = $P_{busy}(t) = \sum_{n=0}^{\infty} P_{n1}(t)$

c. $A_q(t)$ = mean no. of occupants in the orbit at time t = $\sum_{n=0}^{\infty} n\left(P_{n0}(t) + P_{n1}(t)\right)$

III. NUMERICAL COMPUTATIONS

Transient Probabilities and System Performance Measures have been expressed in the form of Tables for Various Values $\lambda, \mu, \sigma, \delta$ and t.

Table 1, Table 2, Table 3 and Table 4 show Transient probabilities of no. of occupants in the orbit when the server is idle in the system for various values of $M,\ \lambda, \mu, \sigma, \delta$ and t.

Table 5, Table 6, Table 7 and Table 8 show Transient probabilities of no. of occupants in the orbit when the server is busy in the system for various values of M, $\lambda, \mu, \sigma, \delta$ and t.

Table 9, Table 10, Table 11 and Table 12 show Time dependent System performance measures for various values of M, λ, μ, σ and δ.

We infer:

1. $A_q(t)$ increases as arrival rate λ increases for all values of t.
2. The value of t increases and for various values of M, λ, μ, σ and δ. $P_{idle}(t) \to P_{idle}, P_{busy}(t) \to P_{busy}, A_q(t) \to A_q$

Table 1: Transient probability distribution of no. of occupants in the orbit when the server is idle in the system for λ =5, μ =10, σ = 5, δ = 3, M = 5 and various values of t.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$	$P_{30}(t)$	$P_{40}(t)$
0.2000	0.0771	0.1027	0.0745	0.0277	0.0042
0.4000	0.0279	0.0838	0.1024	0.0585	0.0130
0.6000	0.0201	0.0747	0.1063	0.0681	0.0165
0.8000	0.0184	0.0723	0.1069	0.0706	0.0175
1.0000	0.0180	0.0716	0.1071	0.0712	0.0178
1.2000	0.0179	0.0715	0.1071	0.0714	0.0178
1.4000	0.0179	0.0714	0.1071	0.0714	0.0179
1.6000	0.0179	0.0714	0.1071	0.0714	0.0179
1.8000	0.0179	0.0714	0.1071	0.0714	0.0179
2.0000	0.0179	0.0714	0.1071	0.0714	0.0179

Table 2: Transient probability distribution of no.of occupants in the orbit when the server is idle in the system for λ =7, μ =10, σ = 5, δ = 3, M = 5 and various values of t.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$	$P_{30}(t)$	$P_{40}(t)$
0.2000	0.0261	0.0676	0.0833	0.0506	0.0124
0.4000	0.0074	0.0412	0.0872	0.0825	0.0294
0.6000	0.0056	0.0361	0.0853	0.0884	0.0337
0.8000	0.0054	0.0352	0.0849	0.0894	0.0345
1.0000	0.0053	0.0350	0.0848	0.0895	0.0347
1.2000	0.0053	0.0350	0.0848	0.0896	0.0347
1.4000	0.0053	0.0350	0.0848	0.0896	0.0347
1.6000	0.0053	0.0350	0.0848	0.0896	0.0347
1.8000	0.0053	0.0350	0.0848	0.0896	0.0347
2.0000	0.0053	0.0350	0.0848	0.0896	0.0347

Table 3: Transient probability distribution of no. of occupants in the orbit when the server is idle in the system for λ =5, μ =10, σ = 5, δ = 3, M = 7and various values of t.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$	$P_{30}(t)$	$P_{40}(t)$	$P_{50}(t)$	$P_{60}(t)$
0.2000	0.0221	0.0515	0.0662	0.0512	0.0242	0.0064	0.0008
0.4000	0.0045	0.0235	0.0538	0.0679	0.0495	0.0197	0.0033
0.6000	0.0028	0.0178	0.0477	0.0688	0.0561	0.0246	0.0045
0.8000	0.0025	0.0165	0.0461	0.0687	0.0577	0.0259	0.0048
1.0000	0.0024	0.0163	0.0457	0.0687	0.0580	0.0262	0.0049
1.2000	0.0024	0.0162	0.0457	0.0687	0.0581	0.0262	0.0049
1.4000	0.0024	0.0162	0.0456	0.0687	0.0582	0.0263	0.0049
1.6000	0.0024	0.0162	0.0456	0.0687	0.0582	0.0263	0.0049
1.8000	0.0024	0.0162	0.0456	0.0687	0.0582	0.0263	0.0049
2.0000	0.0024	0.0162	0.0456	0.0687	0.0582	0.0263	0.0049

Table 4: Transient probability distribution of no. of occupants in the orbit when the server is idle in the system for λ =7, μ = 10, σ = 5, δ = 3, M = 7and various values of t.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$	$P_{30}(t)$	$P_{40}(t)$	$P_{50}(t)$	$P_{60}(t)$
0.2000	0.0044	0.0197	0.0435	0.0557	0.0425	0.0180	0.0033
0.4000	0.0007	0.0063	0.0247	0.0518	0.0612	0.0386	0.0101
0.6000	0.0005	0.0049	0.0214	0.0493	0.0633	0.0429	0.0120
0.8000	0.0004	0.0047	0.0209	0.0489	0.0636	0.0436	0.0123
1.0000	0.0004	0.0047	0.0208	0.0488	0.0637	0.0438	0.0124
1.2000	0.0004	0.0047	0.0208	0.0488	0.0637	0.0438	0.0124
1.4000	0.0004	0.0047	0.0208	0.0488	0.0637	0.0438	0.0124
1.6000	0.0004	0.0047	0.0208	0.0488	0.0637	0.0438	0.0124
1.8000	0.0004	0.0047	0.0208	0.0488	0.0637	0.0438	0.0124
2.0000	0.0004	0.0047	0.0208	0.0488	0.0637	0.0438	0.0124

Table 5: Transient probability distribution of no. of occupants in the orbit when the server is busy in the system for λ =5, μ =10, σ = 5, δ = 3, M =5 and various values of t.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$
0.2000	0.1335	0.2543	0.2180	0.0921	0.0157
0.4000	0.0618	0.2018	0.2604	0.1548	0.0355
0.6000	0.0486	0.1846	0.2664	0.1725	0.0422
0.8000	0.0456	0.1801	0.2675	0.1770	0.0440
1.0000	0.0449	0.1790	0.2678	0.1782	0.0445
1.2000	0.0447	0.1787	0.2678	0.1785	0.0446
1.4000	0.0447	0.1786	0.2679	0.1785	0.0446
1.6000	0.0446	0.1786	0.2679	0.1786	0.0446
1.8000	0.0446	0.1786	0.2679	0.1786	0.0446
2.0000	0.0446	0.1786	0.2679	0.1786	0.0446

Table 6: Transient probability distribution of no. of occupants in the orbit when the server is busy in the system for $\lambda =7, \mu =10, \sigma = 5, \delta =3, M =5$ and various values of t.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$
0.2000	0.0631	0.2010	0.2728	0.1775	0.0457
0.4000	0.0239	0.1306	0.2687	0.2455	0.0836
0.6000	0.0195	0.1181	0.2641	0.2573	0.0919
0.8000	0.0188	0.1160	0.2632	0.2593	0.0934
1.0000	0.0187	0.1156	0.2630	0.2597	0.0936
1.2000	0.0186	0.1155	0.2630	0.2597	0.0937
1.4000	0.0186	0.1155	0.2630	0.2598	0.0937
1.6000	0.0186	0.1155	0.2630	0.2598	0.0937
1.8000	0.0186	0.1155	0.2630	0.2598	0.0937
2.0000	0.0186	0.1155	0.2630	0.2598	0.0937

Table 7: Transient probability distribution of no. of occupants in the orbit when the server is busy in the system for $\lambda = 5, \mu = 10, \sigma =5, \delta =3, M =7$and various values of t.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$
0.2000	0.0502	0.1552	0.2318	0.2015	0.1048	0.0304	0.0038
0.4000	0.0138	0.0766	0.1830	0.2392	0.1795	0.0731	0.0126
0.6000	0.0094	0.0610	0.1655	0.2407	0.1979	0.0872	0.0161
0.8000	0.0086	0.0576	0.1611	0.2405	0.2022	0.0908	0.0170
1.0000	0.0084	0.0568	0.1600	0.2404	0.2032	0.0917	0.0172
1.2000	0.0084	0.0566	0.1598	0.2404	0.2035	0.0919	0.0173
1.4000	0.0084	0.0566	0.1597	0.2404	0.2035	0.0919	0.0173
1.6000	0.0084	0.0566	0.1597	0.2404	0.2035	0.0919	0.0173
1.8000	0.0084	0.0566	0.1597	0.2404	0.2036	0.0919	0.0173
2.0000	0.0084	0.0566	0.1597	0.2404	0.2036	0.0919	0.0173

Table 8: Transient probability distribution of no. of occupants in the orbit when the server is busy in the system for $\lambda =7, \mu =10, \sigma =5, \delta = 3, M = 7$and various values of t.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$
0.2000	0.0141	0.0743	0.1792	0.2448	0.1967	0.0872	0.0165
0.4000	0.0030	0.0280	0.1075	0.2203	0.2534	0.1550	0.0393
0.6000	0.0023	0.0229	0.0959	0.2117	0.2600	0.1682	0.0447
0.8000	0.0021	0.0221	0.0940	0.2102	0.2610	0.1704	0.0456
1.0000	0.0021	0.0220	0.0937	0.2100	0.2611	0.1707	0.0458
1.2000	0.0021	0.0220	0.0936	0.2099	0.2612	0.1708	0.0458
1.4000	0.0021	0.0220	0.0936	0.2099	0.2612	0.1708	0.0458
1.6000	0.0021	0.0220	0.0936	0.2099	0.2612	0.1708	0.0458
1.8000	0.0021	0.0220	0.0936	0.2099	0.2612	0.1708	0.0458
2.0000	0.0021	0.0220	0.0936	0.2099	0.2612	0.1708	0.0458

Table 9: Time dependent system measures for $\lambda = 5, \mu = 10, \delta = 3, \sigma = 5, M = 5$ and various values of t

t	$P_{idle}(t)$	$P_{busy}(t)$	$A_q(t)$
0.2000	0.2864	0.7136	2.0950
0.4000	0.2857	0.7143	2.5595
0.6000	0.2857	0.7143	2.6756
0.8000	0.2857	0.7143	2.7046
1.0000	0.2857	0.7143	2.7119
1.2000	0.2857	0.7143	2.7137
1.4000	0.2857	0.7143	2.7141
1.6000	0.2857	0.7143	2.7142
1.8000	0.2857	0.7143	2.7143
2.0000	0.2857	0.7143	2.7143

Table 10: Time dependent system measures for $\lambda = 7, \mu = 10, \delta = 3, \sigma = 5, M = 5$ and various values of t

t	$P_{idle}(t)$	$P_{busy}(t)$	$A_q(t)$
0.2000	0.2399	0.7601	2.6575
0.4000	0.2477	0.7523	3.0717
0.6000	0.2491	0.7509	3.1432
0.8000	0.2494	0.7506	3.1556
1.0000	0.2494	0.7506	3.1578
1.2000	0.2494	0.7506	3.1582
1.4000	0.2494	0.7506	3.1582
1.6000	0.2494	0.7506	3.1583
1.8000	0.2494	0.7506	3.1583
2.0000	0.2494	0.7506	3.1583

Table 11: Time dependent system measures for $\lambda = 5, \mu = 10, \delta = 3, \sigma = 5, M = 7$ and various values of t

t	$P_{idle}(t)$	$P_{busy}(t)$	$A_q(t)$
0.2000	0.2223	0.7777	3.0660
0.4000	0.2222	0.7778	3.7482
0.6000	0.2222	0.7778	3.9096
0.8000	0.2222	0.7778	3.9478
1.0000	0.2222	0.7778	3.9568
1.2000	0.2222	0.7778	3.9589
1.4000	0.2222	0.7778	3.9594
1.6000	0.2222	0.7778	3.9596
1.8000	0.2222	0.7778	3.9596
2.0000	0.2222	0.7778	3.9596

Table 12: Time dependent system measures for $\lambda = 7\ \mu = 10, \delta = 3, \sigma = 5, M = 7$ and various values of t

t	$P_{idle}(t)$	$P_{busy}(t)$	$A_q(t)$
0.2000	0.1872	0.8128	3.8556
0.4000	0.1933	0.8067	4.4448
0.6000	0.1944	0.8056	4.5400
0.8000	0.1946	0.8054	4.5554
1.0000	0.1946	0.8054	4.5580
1.2000	0.1946	0.8054	4.5584
1.4000	0.1946	0.8054	4.5584
1.6000	0.1946	0.8054	4.5584
1.8000	0.1946	0.8054	4.5584
2.0000	0.1946	0.8054	4.5584

IV. Conclusion

A new computational approach was used to evaluate the Transient behaviour of finite source Retrial queueing model with reneging customers using Eigen values and vectors and infinitesimal generator matrix. In this model we have provided transient probability distribution of number of customers in the orbit at time t and also time dependent system measures.

REFERENCES

[1] Artalejo. J.R (2010), Accessible bibliography on retrial queues Progress in 2000-2009, *Mathematical and Computer Modelling, Vol 51, pp 1071-1081*.

[2] Falin G.I (1990),A survey of retrial queues, *Queueing Systems 7,No.2,127-167*

[3] Falin G.I and J.G.C. Templeton (1997),Retrial queues, *Chapman and Hall, London*

[4] M.F. Neuts (1981),Matrix Geometric Solutions in Stochastic Models-An algorithmic Approach, *The John Hopkins University Press, Baltimore*

[5] Parthasarathy and Sudhesh (2008), Transient solution of a multiserver Poisson queue with N- policy, *Computers and Mathematics with Applications, Vol. 55, pp. 550-562.*

[6] Patrick Wüchner, János Sztrik, Hermannde Meer (2009)," Finite-source M/M/S retrial queue with search for balking and impatient customers from the orbit", *Computer Networks, Volume 53, Issue 8, 11 June 2009, Pages 1264-1273.*

[7] Sudhesh (2010), Transient analysis of queue with system disaster and occupants impatience , *Queueing systems , Vol 66(1), 95105*

[8] Sherif Ammar, (2015), Transient analysis of an M/M/1 queue with impatient behaviour and Multiple vacation, *applied Mathematics and Computations, Vol 260,97-105*

[9] Sherif Ammar (2017), Transient solution of M/M/1 vacation queue with a waiting server and impatient occupants, *Journal of Egyptian Mathematical Society, Vol. 25, 337-342.*

[10] Sophia.S (2016)," Transient Analysis of a Discouraged Arrival Queue subject To Total Catastrophes, Failures And Repairs ", *journal of Pure and Applied Mathematics, Volume 108 No. 1 2016, 93-106; ISSN: 1314-3395 .*

[11] Yang T and Templeton J.G.C (1987). A Survey on retrial queues. *Queueing systems, 2, No. 201-233.*

[12] Rakesh Kumar, Sapana Sharma (2018)," Transient Performance Analysis Of A Single Server Queuing Model With Retention Of n Reneging Customers*" Yugoslav Journal Of Operations Research, Number 3, 315–331.*