Control Chart using ZubairRayleigh Distribution

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Abstract

The frequency distributions of the observed process data, monitoring the control charts, and quality attributes are the major components of the statistical process control methodology. The major issue with quality processes or outcomes is figuring out which alterations should be taken into account to raise the caliber of products. One will not be able to appropriately modify the process if they do not fully comprehend the sources of process variability or result measurement in a complex system. To evaluate the procedure' effectiveness in this paper, control charts were created using the Zubair Rayleigh distribution.

***Keywords**: Statistical Quality control, Control charts, Zubair Rayleigh distribution*

1. Introduction

The production process's variation affects the product's quality. This process variance could be caused by both controllable and uncontrollable causes. Control charts are crucial tools for ensuring the products are of a high calibre. Professor Shewhart Walter established control charts in the 1920s, and they have since developed into a crucial tool in quality improvement. Control charts are used to keep an eye on a process while a product is being manufactured. Based on control charts, prompt action should be taken regarding the process; that is, when a control chart analysis demonstrates that the process is under control, no action should be taken. To put the process back under control, however, appropriate action should be done if the control chart indicates that it has shifted. Consequently, a control chart suggests the appropriate observation or time for corrective action. The upper control limit (UCL) and lower control limit (LCL) are two control limits that are alternately employed to monitor the process and boost industry profits. Control charts used to maintain quality help an industry gain a positive reputation in the marketplace. When the quality of interest follows a normal distribution, Shewhart control charts are frequently employed in the area to monitor the process. In reality, the variable of interest may not necessarily follow the distribution but instead may follow non-normal distributions. The following is the organizing of this paper. The relevant literature is reviewed in section 2. The Zubair Rayleigh distribution is presented in Section 3. Information on control restrictions is provided in Section 4. The section explains the method of identifying out of control using actual data. Summary is presented in Section 5.

Review of literature

Control Charts is a regularly used methodology to improve processes in modern industrial and service industries. This methodology mainly uses control charts and frequency distributions of process and quality attribute data. The normal distribution of the quantitative trait of interest is assumed when creating control charts, although in reality, this is not always the case. The relevant variable might have some non-normal distributions. Statistical Quality Control Application and the Zubair Gumbel distribution have recently been compared, according to Muthusamy and Venkatesan (2022). Using the Zubair Uniform distribution with Statistical Quality Control Application, Muthusamy and Venkatesan (2022). Amin and Venkatesan (2017) recommended comparing the traditional charts and the Bayesian approach to look for minute variations in the control charts. Additionally, they recently examined the developments in control chart techniques in 2019. Derya and Canan (2012) developed standard control charts based on the weibull, lognormal, and exponential distributions to transform non-normal data into calculations for process control and process capability. Santiago and Smith made the most recent proposal for the control charts (2013). For the Mukherjee and Islam distribution, which they introduced, the exponentiated distribution technique is applied (1983). They used Nelson's (1994) variable transformation to make exponentially dispersed data somewhat resemble normal data. the application of charts that can be utilised as a model for process optimization. Rao (1965) formulated it and articulated it extensively in connection with the modelling of statistical data when it was demonstrated that the conventional practise of employing standard distributions was incorrect. When Zubair Rayleigh distributions were being utilised to change practise, it was improper to employ standard distributions. The Zubair Rayleigh distribution can be used to jointly access the model definition and date interpretation difficulties. The Zubair Rayleigh distributions take into account the method of ascertainment by modifying the probability of events taking place in reality to arrive at a specification of the probabilities of those events as observed and recorded. The Zubair Rayleigh distribution has been used to build a number of distributions to track the process.

3. Description of the distribution

Definition: A random variable X has a Zubair Rayleigh distribution if the density function is given by

$$f\left(x,\alpha,\sigma^2\right)=\frac{2\alpha\frac{x}{\sigma^2}e^{\frac{-x^2}{2\sigma^2}}\left(1-e^{\frac{-x^2}{2\sigma^2}}\right)\exp\left\{\alpha\left(1-e^{-\frac{x^2}{2\sigma^2}}\right)^2\right\}}{\exp(\alpha)-1}\quad x>0,\alpha>0,\sigma^2>0 \qquad \textbf{(1)}$$

Where α and σ^2 are the shape and scale parameters of the distribution respectively. The moment generating function can be obtained by

$$M_X(t)=\int_{-\infty}^{\infty}e^{tx}f(x)dx$$

$$=\int_{-\infty}^{\infty}\frac{2\alpha}{\sigma^2}e^{tx}\frac{x e^{t}\left(1-e^{t}\right)e^{\alpha}\left(1-e^{t}\right)^2}{e^{\alpha}-1}dx$$

One gets after simplifications

$$M_X(t)=\frac{2\alpha}{\left(e^{\alpha}-1\right)\left(1+x\right)\left(2+x\right)} \qquad \textbf{(2)}$$

In order to use this distribution, one need study the mean and variance of the distribution. The mean variance are obtained are respectively are given as follows,

$$E(X)=\frac{\alpha}{\left(e^{\alpha}-1\right)} \qquad \textbf{(3)}$$

$$V\left(X\right) = \frac{\alpha\left\{7\left(e^{\alpha} - \right)^{2} - 4\alpha\right\}}{4\left(e^{\alpha} - 1\right)} \qquad (4)$$

4. Control limits

Three sigma UCL and LCL are obtained as specified by Montgomery (2012) and one can get the control limits for Zubair Rayleigh distribution using (3) and (4) and are given by

$$UCL = \frac{\alpha}{\left(e^{\alpha} - 1\right)} + 3\sqrt{\frac{\alpha\left[7\left(e^{\alpha} - 1\right)^{2} - 4\alpha\right]}{4\left(e^{\alpha} - 1\right)}}$$

$$CL = \frac{\alpha}{\left(e^{\alpha} - 1\right)}$$

$$LCL = \frac{\alpha}{\left(e^{\alpha} - 1\right)} - 3\sqrt{\frac{\alpha\left[7\left(e^{\alpha} - 1\right)^{2} - 4\alpha\right]}{4\left(e^{\alpha} - 1\right)}}$$

5. Numerical Illustration

To demonstrate the uses of the suggested strategy, a case study involving the development of control limits is taken into consideration. With the help of a simulated data set for the parameters, the control limits of the Zubair Rayleigh distribution are discovered. Table 1 reports all of the samples that were generated.

Data Set:

Management at Webster, in problem is now concerned as to whether caulking tubes are being properly capped. If a significant proportion of the tubes are not being sealed, Webster is placing its customers in a messy situation. Tubes are packaged in large box of 144. Several boxes are inspected, and the following numbers of leaking tubes are found. Calculate p chart three- sigma control limits to assess Whether the capping process is in statistical control.

3,	5,	3,	10,	8,	4,	12,	6,	4,	13,	2,10,	
5,	12,	7,	9,	14,	7,	9,	13,	6,	11,	3,	4.

Table 1: Control limits Using Zubair Rayleigh Distribution

α	UCL	CL	LCL
1	7.37	0.58	-6.71
2	14.41	0.31	-13.79
3	30.16	0.16	-29.84
4	58.18	0.07	-58.04
5	107.76	0.03	-107.70
6	195.01	0.01	-194.99
7	347.4	0	-347.4
8	1667.4	0	-1667.4
9	1071.6	0	-1071.6
10	1860	0	-1860

It is observed from the Table1, that for the fixed value of the parameter α , with a as random variable that target valuethe Zubair Rayleigh control chart is fig.1and$\alpha = 2$

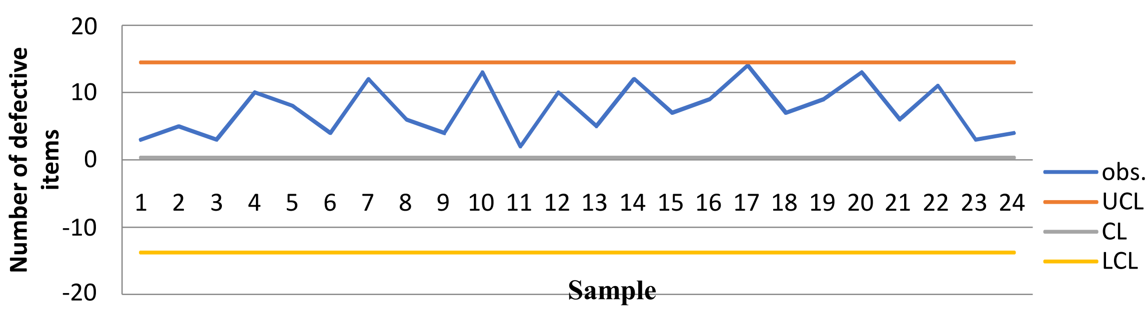

Fig.1. P chart

Additionally, it has been noted that for a process to remain under control, its observations must be $0<X<\alpha$;$\alpha > 0$;$\sigma^2>0$ In other words, depending on the products being manufactured, the manufacturing engineers should fix the parameter values based on the kind of data they are using. Table 1 shows this, as can be seen. The increasing of parameterα .

6. Conclusions

For process quality control in this investigation, a Zubair Rayleigh distribution is employed. The control limits for the Zubair Rayleigh distribution are also determined when and have different values. In order to help the manufacturing engineer choose the parameters based on the kind of data they are working with, a table is prepared. The special case where a=2 and all observations are shown in control is taken into consideration while creating the control chart. It is recommended to keep more process data than what is noticed as a result. Hence it is advisable to keep greater than the observed process data.

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