Comparative Analysis of Radon Concentration in Air and Charcoal Medium through Time Fractional Radon Diffusion Equation .

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Abstract*:* This research paper discusses the comparative analysis of Radon Diffusion and concentration measurements in two different mediums. Focus is given on Charcoal and air medium for the investigation on Radon transport and diffusion. The parabolic Radon diffusion equation which is a basic partial differential equation, is further converted into time fractional diffusion equation. The fractional differential equation is further analysed for its solution using Implicit finite difference scheme. The analytical estimations for various physical and chemical parameters related to Radon are connected with the diffusion equation and computations for the two mediums are done separately. Further the numerical computations and graphical representations are compared with reference the solution. The Stability and the convergence approach has also been discussed for the two different schemes.

Keywords: *Time fractional differential equation, Caputo fractional derivative, diffusion equation, Implicit method ,stability ,convergence*

2010 Classifications: 35R10 Partial functional-differential equations ; 65M06 Finite difference methods

i. INTRODUCTION

Diffusion basically is the phenomena of spread of a matter through some medium due to inherent molecular movements. The rate of diffusion in various mediums is different. It is basically dependent on time, temperature, type of the substance, atomic structure of the substance, atmospheric conditions and many other physical and chemical properties of the substance. Gas diffusion in other gaseous mediums is comparatively faster as the molecular distance is more and the molecular size is smaller. [1],[2] have discussed in their research work about diffusion of Radon, the noble radioactive gas in the activated charcoal canister. The loss of concentration level of Radon due to radioactive properties is studied with its formulated model of diffusion equation which is a partial differential equation. The Basic diffusion equation with decay of radon been added is the formulated structure of Radon diffusion equation, $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} - \lambda u$. Further we convert this equation into time fractional form and solve it by using finite difference scheme. $u(x,t)$ is the radon concentration as function of time and space [12],[13].

As Radon gas has normal characteristic of spread, it has always been a reality of inquisitive study. Detection of radon in different category of civil structures and resources where it is found causing ill effects due to its spread to living beings.

The aim of this research paper hence is to :

1. To obtain solution to time fractional Radon diffusion equation..
2. To implement implicit finite difference scheme for solving the equation.
3. To find the analytical estimates in air medium.
4. To find the analytical estimates in charcoal medium.
5. To compare the results obtained in the two different schemes.
6. To discuss the results about the stability and convergence for the solutions.

We have applied implicit finite difference scheme to solve the time fractional diffusion equation in this paper. The measurements for various parameters and estimates have been referred from the reference of master thesis by Rybalkin, Andrey (2012)[10], "Numerical and analytical assessment of radon diffusion in various media and potential of charcoal as radon detector" .

The paper is divided in 9 sections, as follows: 1) Introduction 2) FDM and the solution to TFRDE.3) Analytical estimates in air medium, Analytical estimates in charcoal medium 4) Stability of the solution 5) convergence of the solution 6) Graphical interpretation of solution 7) Results and discussion 10) References

ii. RADON DIFFUSION EQUATION AND IMPLICIT FINITE DIFFERENCE SCHEME

For measuring the radon concentration, we study the diffusion through air medium. The second order equation which is the focus for this paper, has been solved using the implicit finite difference scheme.

$$\frac{\partial^\alpha \mathrm{v(x,t)}}{\partial \mathrm{t}^\alpha} = \mathrm{D}\frac{\partial^2 \mathrm{v(x,t)}}{\partial \mathrm{x}^2} - \lambda \mathrm{v(x,t)}, \text{where } \lambda \text{ is decay constant of radon.}$$

We consider the following equation which is time fractional diffusion equation,

$$\frac{\partial^\alpha \mathrm{v(x,t)}}{\partial \mathrm{t}^\alpha} = \mathrm{D}\frac{\partial^2 \mathrm{v(x,t)}}{\partial \mathrm{x}^2} - \lambda \mathrm{v(x,t)} \qquad (2.1)$$

$$\text{IC: } \mathrm{v(x,0)} = 0, 0 < x < l \qquad (2.2)$$

$$\text{B.C.}: v(0,\mathrm{t}) = \mathrm{v}_0, \mathrm{t} \geq 0 \text{ and } \frac{\partial \mathrm{v(x,t)}}{\partial \mathrm{x}} = 0, \mathrm{t} \geq 0 . \quad 0 \leq \alpha \leq 1 \qquad (2.3)$$

iii. DISCRITIZATION SCHEME

To covert the time fractional derivative in discrete form, we use $t_k = k\tau$, and $x_i = ih, \tau = \frac{T}{N}, h = \frac{l}{\mathrm{N}}$. . Let $\mathrm{v}(x_i, t_k), i = o, 1,2 \ldots\ldots, M \text{ and } k = o, 1,2, \ldots\ldots, N$ be the exact solution of TFRDE from (2.1) -(2.3) at the mesh point (x_i, t_k). Let u_i^k be the numerical approximation of the point $\mathrm{v}(x_i, t_k) = \mathrm{v}(ih, k\tau)$.

The time fractional derivative is approximated in Caputo sense is given by,

$$\frac{\partial^\alpha \mathrm{v}(x_i, t_{k+1})}{\partial \mathrm{t}^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^{t_{k+1}} \frac{1}{(t_{k+1} - \xi)^\alpha} \frac{\partial \mathrm{v}(x_i, \xi)}{\partial \xi} \mathrm{d}\xi$$

Substitute $t_{k+1} - \xi = \eta$ and simplifying we get

$$= \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \sum_{\mathrm{j}=0}^{\mathrm{k}} \frac{\left(\mathrm{v}(x_i, t_{k-j+1}) - v(x_i, t_{k-j})\right)}{\tau} [\, \mathrm{b_j}] + \mathrm{o}(\tau);$$

where, $\mathrm{b_j} = (\mathrm{j}+1)^{(1-\alpha)} - (\mathrm{j})^{(1-\alpha)}, \mathrm{j} = 0,1,2 \ldots \mathrm{N}$ but $\mathrm{b}_0 = 1$, so we have;

$$\frac{\partial^\alpha \mathrm{v}(x_i, t_{k+1})}{\partial \mathrm{t}^\alpha} == \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} [\mathrm{v_i^{k+1}} - \mathrm{v_i^k} [\, \mathrm{b}_0] + \mathrm{o}(\tau) + \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \sum_{\mathrm{j}=1}^{\mathrm{k}} \frac{\left(\mathrm{v_i^{k-j+1}} - \mathrm{v_i^{k-j}}\right)}{\tau} [\, \mathrm{b_j}]$$

For approximating the second order space derivative, we adopt a symmetric second order difference quotient in space at level $t = t_{k+1}, i.e.$

$$\frac{\partial^2 \mathrm{v(x,t)}}{\partial \mathrm{x}^2} = [\frac{\mathrm{v_{i-1}^{k+1}} - 2v_i^{k+1} + v_{i+1}^{k+1})}{\mathrm{h}^2}]$$

So the numerical approximation equation thus obtained is

$$\frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}[\mathrm{v}_i^{k+1}-\mathrm{v}_i^{k}]+\frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\sum_{j=1}^{k}[\mathrm{b}_j]\left(\mathrm{v}_i^{k-j+1}-\mathrm{v}_i^{k-j}\right)=\mathrm{D}.\left[\frac{\mathrm{v}_{i-1}^{k+1}-2v_i^{k+1}+v_{i+1}^{k+1})}{\mathrm{h}^2}\right]-\lambda\mathrm{v}(x_i,t_k)$$

Let $r=\mathrm{D}\ \frac{\Gamma(2-\alpha)\tau^{\alpha}}{\mathrm{h}^2}$ *and* $\mu=\Gamma(2-\alpha)\,\lambda$

$$-\mathrm{rv}_{i-1}^{k+1}+(1+2\mathrm{r})\mathrm{v}_i^{k+1}-\mathrm{rv}_{i+1}^{k+1}=(1-\mu-\mathrm{b}_1)\mathrm{v}_i^{k}+\sum_{j=1}^{k-1}(\mathrm{b}_j-\mathrm{b}_{j-1})\,\mathrm{v}_i^{k-j}+\mathrm{b}_k\mathrm{v}_i^{0}\ \ldots.(2.5)$$

where , $\mathrm{b}_j=(j+1)^{1-\alpha}-(j)^{1-\alpha}$; $i=0,1,2.\ldots.m$; $k=0,1,2,\ldots.n$

Now we convert the initial condition and boundary conditions in discritized format:
$\mathrm{v}_i^0=0$; $\mathrm{i}=0,1,2\ldots.\mathrm{m}$;
The boundary conditions $x_{0,}$and $x_{m,}$the discritization scheme implements as:

$$\mathrm{v}_0^k=0\ \text{and}\ \frac{\mathrm{v}_{m+1}^{k+1}-\mathrm{v}_{m-1}^{k+1}}{2\mathrm{h}}=0\ ;\text{implies}\ \ \mathrm{v}_{m+1}^{k+1}=\mathrm{v}_{m-1}^{k+1}$$

For $k=0$, we have,

$$\mathrm{v}_i^1-\mathrm{v}_i^0=\mathrm{r}[\mathrm{v}_{i-1}^1-2\mathrm{v}+\mathrm{v}_{i+1}^1]-(\mu)\mathrm{v}_i^0$$

$-r\mathrm{v}_{i-1}^1+(1+2\mathrm{r})\mathrm{v}_i^1-\mathrm{rv}_{i+1}^1=(1-\mu)\mathrm{v}_i^0$ ....

Therefore the complete discredited IBVP is ,
$-r\mathrm{v}_{i-1}^1+(1+2\mathrm{r})\mathrm{v}-\mathrm{rv}_{i+1}^1=(1-\mu)\mathrm{v}_i^0$ .... (2.6)

$$-\mathrm{rv}_{i-1}^{k+1}+(1+2\mathrm{r})\mathrm{v}_i^{k+1}-\mathrm{rv}_{i+1}^{k+1}=(1-\mu-\mathrm{b}_1)\mathrm{v}_i^{k}+\sum_{j=1}^{k-1}(\mathrm{b}_j-\mathrm{b}_{j+1})\mathrm{v}_i^{k-j}+\mathrm{b}_k\mathrm{v}_i^{0}\ (k\geq 1)\ \ldots.(2.7)$$

With initial conditions , $\mathrm{v}_i^0=0,\mathrm{i}=0,1,2\ldots\ldots\mathrm{m}$ .... (2.8)
$\mathrm{v}_{m+1}^{k+1}=\mathrm{v}_{m-1}^{k+1}$ (2.9)
The problem (2.6) to (2.9) is the complete discritized form of (2.1) to (2.3)

So, the equation now converted to matrix form becomes at $k=0$ *and* $i=1,2,3\ldots m$

$$-r\mathrm{v}_{i-1}^1+(1+2\mathrm{r})\mathrm{v}_i^1-\mathrm{rv}_{i+1}^1=(1-\mu)\mathrm{v}_i^0$$

Can be represented in matrix form as;

$$\begin{bmatrix}(1+2r) & -r & \cdot\ \cdot & & \cdot\ \cdot & \\ -r & (1+2r) & \cdot\ \cdot & & \cdot\ \cdot & \\ \cdot\ \cdot & & \cdot\ \cdot & & \cdot\ \cdot & \\ \cdot\ \cdot & & \cdot\ \cdot & & \cdot\ \cdot & \\ \cdot\ \cdot & & \cdot\ \cdot & (1+2r) & & -r \\ \cdot\ \cdot & & \cdot\ \cdot & -2r & & (1+2r)\end{bmatrix}\begin{bmatrix}v_1^1\\ v_2^1\\ \cdot\\ v_{m-1}^1\\ v_m^1\end{bmatrix}=(1-\mu)\begin{bmatrix}v_1^0\\ v_2^0\\ \cdot\\ v_{m-1}^0\\ v_m^0\end{bmatrix}+\begin{bmatrix}rv_0^1\\ 0\\ \cdot\\ 0\\ 0\end{bmatrix}\ldots(2.10)$$

$\mathrm{Av}^1=(1-\mu)\mathrm{v}^0+\mathrm{E}$ (2.10)

Now for *for* $k\geq 1$;

$$-\mathrm{rv}_{i-1}^{k+1}+(1+2\mathrm{r})\mathrm{v}_i^{k+1}-\mathrm{rv}_{i+1}^{k+1}=(1-\mu-\mathrm{b}_1)\mathrm{v}_i^{k}+\sum_{j=1}^{k-1}(\mathrm{b}_j-\mathrm{b}_{j+1})\mathrm{v}_i^{k-j}+\mathrm{b}_k\mathrm{v}_i^{0}\ (i=1,2,3..m)$$

The matrix representation is given by,

$$\begin{bmatrix}(1+2r) & -r & \cdot\ \cdot & & \cdot\ \cdot & \\ -r & (1+2r) & \cdot\ \cdot & & \cdot\ \cdot & \\ \cdot\ \cdot & & \cdot\ \cdot & & \cdot\ \cdot & \\ \cdot\ \cdot & & \cdot\ \cdot & & \cdot\ \cdot & \\ \cdot\ \cdot & & \cdot\ \cdot & (1+2r) & & -r \\ \cdot\ \cdot & & \cdot\ \cdot & -2r & & (1+2r)\end{bmatrix}\begin{bmatrix}v_1^{k+1}\\ v_2^{k+1}\\ \cdot\\ v_{m-1}^{k+1}\\ v_m^{k+1}\end{bmatrix}=(\mathrm{b}_k)\begin{bmatrix}v_1^0\\ v_2^0\\ \cdot\\ v_{m-1}^0\\ v_m^0\end{bmatrix}+\begin{bmatrix}rv_0^1\\ 0\\ \cdot\\ 0\\ 0\end{bmatrix}$$

$$+\sum_{j=1}^{k-1}(b_j-b_{j+1})\begin{bmatrix}v_1^{k-j}\\ v_2^{k-j}\\ \cdot\\ \cdot\\ v_m^{k-j}\end{bmatrix}+(1-\mu-b_1)\begin{bmatrix}v_1^{k}\\ v_2^{k}\\ \cdot\\ \cdot\\ v_m^{k}\end{bmatrix}+\begin{bmatrix}rv_0^{k+1}\\ 0\\ \cdot\\ 0\\ 0\end{bmatrix}$$

$Av^{k+1}=(1-\mu-b_1)v^k+\sum_{j=1}^{k-1}(b_j-b_{j+1})v_i^{k-j}+b_k v_i^0+F$ .....(2.11) (2.9)

(2.8)and (2.9)with i.c. $v_i^0=0$ and b.c. $v_0^k=v_0, v_{m+1}^k=v_{m-1}^k$,

$k=$ 0,1,2 ..... N represents the completely discritized matrix form of the problem.

IV. THE ANALYTICAL ESTIMATES:

RADON DIFFUSION IN CHARCOAL MEDIUM:

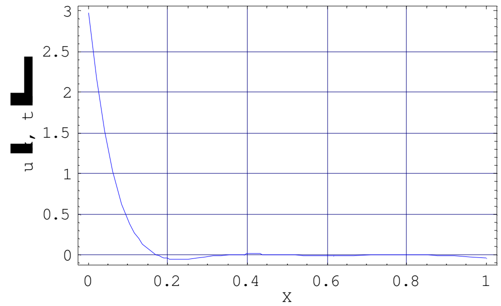
Various parameters for Radon diffusion in Charcoal medium are as follows:

- $l=1.7278\ cm$ canister length
- $s=81cm^2$, surface area of cross section of the cylinder
- $\rho=0.5gm/cm^3$, charcoal density
- $k=4m^3/kg$, activated charcoal adsorption coefficient
- $D=1.43\times10^{-9}m^2/sec$, Radon diffusion coefficient in activated charcoal
- $v_0=200\ Bq/m^3$ Constant radon concentration in air
- $v=1\ m^3$ volume of the cylinder
- $R=\frac{1}{\sqrt{\pi}}\ m$ radius of the cylinder
- $\lambda=2.1\times\frac{10^{-6}}{s}$ is the decay coefficient of Radon.
- $v(0,t)=k\rho v_0$
- $r=\frac{D\times\Gamma(2-\alpha)\times\tau^{\alpha}}{h^2}$
- $\mu=\lambda\,\Gamma(2-\alpha)\tau^{\alpha}$
- $\alpha=$ order of the fractional order derivative.
- $\Delta x=h=0.025$
- $\Delta t=\tau=0.005$
- $v(0,t)=k\rho v_0=\frac{4m^3}{kg}\times\frac{0.5gm}{cm^3}\times\frac{200Bq}{m^3}=0.4\ bq/cm^3$
- $\Gamma(1.1)=0.9513508$ for $\alpha=0.9$, $\Gamma(1.2)=0.9187$for $\alpha=0.8$, $\Gamma(1.3)=0.8947$for $\alpha=0.7$, from gamma tables.

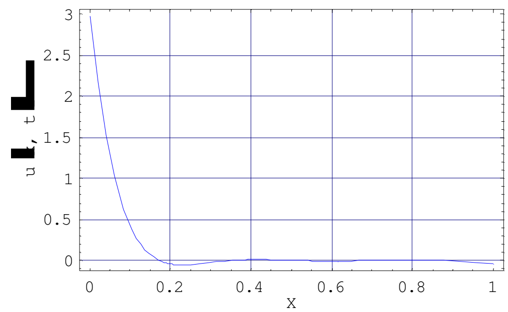
a) For $\alpha=0.9$, $r=1.8487\times10^{-8}$, $\mu=1.6968\times10^{-8}$, $v(0,t)=4\times10^5$, $(1-\mu-b_1)=0.92822$, $b_j-b_{j+1}=0.059722$, $ru_t^0=7.3948\times10^{-8}$

b) For $\alpha=0.8$, $r=3.1403\times10^{-8}$, $\mu=2.78335\times10^{-8}$, $v(0,t)=4\times10^5$, $(1-\mu-b_1)=0.851309$, $b_j-b_{j+1}=0.118189$, $ru_t^0=0.0125612$

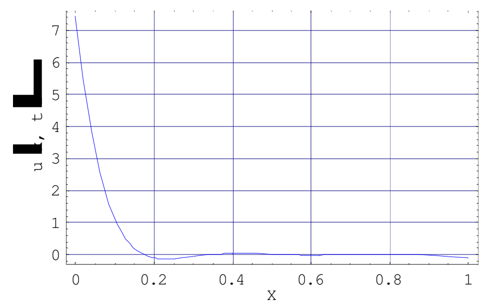
c) For $\alpha=0.9$, $r=5.03216\times10^{-8}$, $\mu=4.6186\times10^{-8}$, $v(0,t)=4\times10^5$, $(1-\mu-b_1)=0.76855$, $b_j-b_{j+1}=0.17653$, $ru_t^0=0.020128$



$Radon\ concentration\ in\ charcoal\ medium\ at\ fractional\ derivative\ \alpha = 0.9$



$Radon\ concentration\ in\ charcoal\ medium\ at\ fractional\ derivative\ \alpha = 0.8$



$Radon\ concentration\ in\ charcoal\ medium\ at\ fractional\ derivative\ \alpha = 0.7$

RADON DIFFUSION IN AIR MEDIUM:

Various parameters for Radon diffusion in air are as follows:

- $D_a = 1 \times \frac{10^{-5} m^2}{s} = 0.1 cm^2/s$ is the diffusion coefficient of radon in air.
- $v_0 = 200 Bq/m^3$ is the radon concentration in ambient air.
- $k = 1m^3/kg\ and\ \rho = 1g/cm^3$ are the radon absorption coefficient
- $l = 1m$ is the length of cylinder for measurement
- $\mathrm{v} = 1m^3$ is the volume of cylinder for measurement , $R = \frac{1}{\sqrt{\pi}} m$ is radius of cylinder used for measurement.

The experiment for measurement of Radon diffusion was conducted for 72 hours duration, for saturation of radon activity in air.

- $\mathrm{v}(0,t) = k\rho C_0 = 1 \times 1 \times 200 = o.2 Bq/cm^3$
- $\lambda = 2.1 \times 10^{-6}/s$ is the decay coefficient of Radon.
- $r = \mathrm{D}\ \frac{\Gamma(2-\alpha)\tau^{\alpha}}{\mathrm{h}^2}$ $and\ \mu = \Gamma(2-\alpha)\ \lambda$, surface area $\mathrm{S} = \pi \mathrm{r}^2$

Let $\alpha = 0.9, 0.8, 0.7$, $\mathrm{b_j} = (\mathrm{j}+1)^{(1-\alpha)} - (\mathrm{j})^{(1-\alpha)}, \mathrm{b_0} = 1$

a) For $\alpha = 0.9, r = 8.08004 * 10^{-6}, S = 1m^2, \mu = 1.696 \times 10^{-8}, So,$

$u(0,t) = 200 * 10^6$

$\mathrm{b_1} = 0.0717,\ \mathrm{b_2} = 0.044, \mathrm{b_3} = 0.03, \mathrm{b_4} = 0.0259, \mathrm{b_5} = 0.0216, \mathrm{b_6} = 0.0185,$
$\mathrm{b_7} = 0.01633, \mathrm{b_8} = 0.01458, \mathrm{b_9} = 0.01319, \mathrm{b_{10}} = 0.01205$

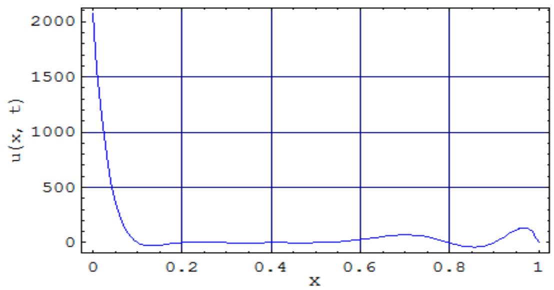
Substituting the calculated values in the equation 2.7 and 2.8 we solved the matrix equations for solutions.

b) $\alpha = 0.8, r = 1.3254 * 10^{-5}, \mu = 0.0278 * 10^{-6}, u(0,t) = 200 * 10^6$

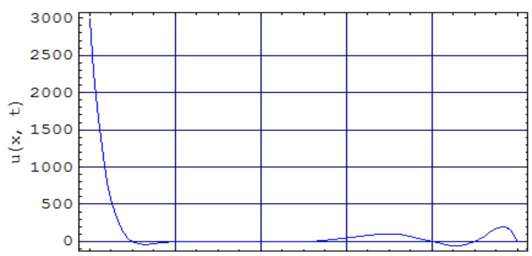
$\mathrm{b_1} = 0.14869,\ \mathrm{b_2} = 0.09703, \mathrm{b_3} = 0.07377, \mathrm{b_4} = 0.060221,$
$\mathrm{b_5} = 0.0512394, \mathrm{b_6} = 0.044804,$
$\mathrm{b_7} = 0.03994, \mathrm{b_8} = 0.036129, \mathrm{b_9} = 0.03304,, \mathrm{b_{10}} = 0.030501$

c) $\alpha = 0.7, r = 2.1993 * 10^{-5}, \mu = 0.04616 * 10^{-6}, u(0,t) = 200 * 10^6$

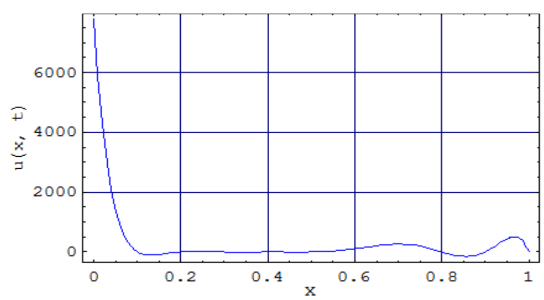
We see the solution interpreted graphically by using 'Mathematica'.



$Radon\ concentration\ in\ air\ medium\ at\ fractional\ derivative\ \alpha = 0.9$



$Radon\ concentration\ in\ air\ medium\ at\ fractional\ derivative\ \alpha = 0.8$



$Radon\ concentration\ in\ air\ medium\ at\ fractional\ derivative\ \alpha = 0.7$

V. STABILITY:

Lemma 4.1: **If $\lambda_j(A)$; $j = 1, 2, 3 \ldots\ldots M-1$, represents equations of matrix A then we prove the following results:**

1. $\left|\lambda_j(A)\right| \geq 1$ **2.** $\|A^{-1}\| \leq 1$, **for** $j = 1,2,3.......M-1$.

The Gresgorian theorem states that each eigen value λ, of matrix A satisfies at least one of the inequalities stated above. **To prove the first statement, we equation (4.3)**, for matrix A, then eigenvalue λ of matrix A satisfies the following inequality, we use method of induction.

Lemma 4.2: The solution obtained for the time fractional radon diffusion equation is unconditionally stable for air medium as well as charcoal medium.

Proof: The stability of the solution obtained for time fractional radon diffusion equation mentioned above, we prove the relation $\|v\|_2 \leq \|v^0\|_2 for\ n \geq 1$. from 2.10, $Av^1 = (1-\mu)v^0 + E$

$$v^1 = A^{-1}(1-\mu)v^0 + A^{-1}E$$

$$\|v^1\|_2 = \|A^{-1}v\|_2(1-\mu) \leq \|A^{-1}\|_2\|v^0\|_2(1-\mu)$$

$$\|v\|_2 \leq k\|v^0\|_2\ E = 0\ and\ \|A^{-1}\|_2 \leq 1, \mu \leq 1$$

By Principle of induction we extend this statement for n=k ;
$\|v\|_2 \leq k\|v^0\|_2$ and for $n = k+1$ from (2.11)

$$Av^{k+1} = (1-\mu-b_1)v^k + \sum_{j=1}^{k-1}(b_j - b_{j+1})\,v_i^{k-j} + vu_i^0 + F$$

$$v^{k+1} = A^{-1}(1-\mu-b_1)v + \sum_{j=1}^{k-1}(b_j - b_{j+1})A^{-1}v_i^{k-j} + A^{-1}b_k v_i^0 + A^{-1}F$$

$$\|v^{k+1}\|_2 \leq |(1-\mu-b_1)|\|A^{-1}\|_2\|v^k\|_2 + \textstyle\sum_{j=1}^{k-1}(b_j - b_{j+1})\|A^{-1}\|_2\left\|v^{k-j}\right\|_2 + |b_k|\|A^{-1}\|_2\|v^0\|_2,$$

$$\|v^{k+1}\|_2 \leq k\|v^0\|_2; (1-\mu) = k$$

These conditions affirm us about the unconditional stability of Implicit finite difference scheme to the Radon diffusion equation.

vi. CONVERGENCE

The convergence of the discritised scheme of approximation with the implicit finite difference scheme towards the exact solution is observed here.

Let $u(x_i,t_k)$ be the exact solution of the time fractional diffusion equation in (2.1) to (2.3) and u_i^k be the exact solution for (2.6) to (2.9) at some point (x_i,t_k) i=1,2,3....m-1; k=1,2,3,.....n. Let $e_i^k = u(x_i,t_k) - u_i^k$. $E^k = (e_1^k, e_2^k \dots\dots.. e_{m-1}^k)$, $E^0 = 0, E_0^k = 0, E_n^k = 0$.

From the discritised scheme

$$-re_{i-1}^1 + (1+2r)e_i^1 - re_{i+1}^1 = (1-\mu)e_i^0; k = 0 \dots\dots (5.1) \quad \text{and}$$

$$-re_{i-1}^{k+1} + (1+2r)e_i^{k+1} - re_{i+1}^{k+1} = (1-\mu-b_1)e_i^k + \sum_{j=1}^{k-1}(b_j - b_{j+1})e_i^{k-j} + b_k e_i^0\ (k \geq 1) \dots.(5.2)$$

$r = D\ \frac{\Gamma(2-\alpha)\tau^\alpha}{h^2}\ and\ \mu = \Gamma(2-\alpha)\,\lambda;\ b_j = (j+1)^{1-\alpha} - (j)^{1-\alpha}$

Lemma 5.1: The fractional order implicit finite difference scheme for the TFRDE u_i^k converges to $u(x_i,t_k)$, the relation between the two solutions satisfies the relation $\left\|u(x_i,t_k) - u_i^k\right\| \leq \|E\|_\infty + O(\tau^{1-\alpha} + h^2), i = 1,2, \dots..m-1; k = 1,2,..n.$

vii. RESULTS AND DISCUSSION:

After solving the time fractional radon diffusion equation in air medium and in charcoal medium (2.1)-(2.3) by applying discritisation technique and Implicit finite difference scheme, we have compared the radon concentration at various lengths of mediums. The approximated solution for time fractional radon diffusion equation in air medium and charcoal medium has been obtained, with initial and boundary conditions. The solutions have been validated numerically by using 'Mathematica' software. We believe the one-dimensional time fractional diffusion equation subjected at fractional derivative of time, α = 0.9,0.8 ,0.7. The numerical solutions are analysed at t = 0.05 by taking into consideration the terms τ = 0.005, h = 0.1, interpreted in the following figures. The comparative table of these two mediums can be seen as follows:

Length x	Radon Concentration Air medium	Length x	Radon Concentration Charcoal medium
0	7776.38	0	7.45229
0.1	1.95559	0.17278	0.000819214
0.2	0.000267862	0.34556	$4.9444049271065685\times 10^{-8}$
0.3	$2.6462489615220867\times 10^{-8}$	0.51834	$2.166871679156984\times 10^{-12}$
0.4	$2.108893009842443\times 10^{-12}$	0.69112	$7.704304651382232\times 10^{-17}$
0.5	$1.4388839164863555\times 10^{-16}$	0.86393	$2.3567461119611746\times 10^{-21}$
0.6	$8.717035130666181\times 10^{-21}$	1.03668	$6.428658318499234\times 10^{-26}$
0.7	$4.8044462729586\times 10^{-25}$	1.20946	$1.601376964811450310^{-30}$
0.8	$2.4508710792138236\times 10^{-29}$	1.38224	$3.7044300993029104\times 10^{-35}$
0.9	$2.3439196711399148\times 10^{-33}$	1.7278	$1.6113820212464783\times 10^{-39}$
1.0	0.0		

viii. CONCLUSION

The numerical solutions are sensed for the diffusion equation to investigate the process of radon transport through air and charcoal. Estimation of values for radon concentration is done in air and charcoal medium for smaller and intermediate diffusion times. The minute difference nullifies as radon concentration tends to its stable condition. Numerical results obtained for the diffusion equation are analysed. It is sure that the fractional order implicit finite difference method offers numerically stable solution. Also the graphs for concentration of radon at various α levels are indicative of the similar rate of concentration up to certain level but little variation can be seen at the tail ends of the graphs. The solution obtained is unconditionally stable in both the mediums. We observe that the numerical solution implicit finite difference method in both mediums are very close. We demonstrated that the solutions obtained in both mediums have second order Convergence. From the comparative analysis speaks about the rate of radon concentration at different levels in charcoal and air medium. The ability to simulate radon diffusion by the method presented enables estimation of radon flux density in buildings.

ix. REFERENCES

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