A Study on the Characteristics of R_1 – Near Ring

S.R.Veronica Valli[1], K.Bala Deepa Arasi[2], D.Radha[3]

[1]*Research Scholar, PG & Research Department of Mathematics,*

A.P.C.Mahalaxmi College for Women, Thoothukudi.

[2,3]*Assistant Professor, PG & Research Department of Mathematics,*

A.P.C.Mahalaxmi College for Women, Thoothukudi.

Abstract

In this paper, we introduce the concept of an R_1 – Near Ring. The properties of R_1 – Near Ring are discussed using the concept of Zero divisors, Idempotents and Sub-Commutativity. On some characteristics of Near Rings, added with homomorphism the resultant is proved to be another substructure of Near Ring. A Pseudo Commutative R_1 – Near Ring has strong IFP, under the existence of identity. Anti-homomorphism preserves the quality of an R_1 – Near Ring whenever it is weak commutative. Every sub-commutative near ring is an R_1 – Near Ring specified to superlative conditions. It is utmost a close correlation for a near ring to be a sub-commutative R_1 – Near Ring. Every β_1 – Near Ring is a β_2 – Near Ring, whenever it is an R_1 – Near Ring. Structure of an R_1 – Near Ring is preserved under certain characterisations.

Keywords: Zero divisors, Pseudo Commutative R_1 – Near Ring, IFP, Anti-homomorphism, β_1 – Near Ring, Weak Commutative, β_2 – Near Ring, Sub-commutative R_1 – Near Ring.

1. Introduction

Near Rings can be thought of as generalised rings: if in a ring we ignore the commutativity of addition and one distributive law, we get a near ring. Gunter Pilz [ ] “Near Rings” is an extensive collection of the work done in the area of near rings. Throughout this paper, N stands for a right near ring with at least two elements and ‘0’ denotes the identity element of the group (N, +) and we write xy for x.y for any two elements x, y of N. Obviously 0n=0 for all n ∈ N then we say that N is zero symmetric. An element a is said to be nilpotent if $a^k = 0$ for some positive integer k. In this paper, we discuss the various characteristics of an R_1 – Near Ring. The terms which are left undefined in this paper can be found in Gunter Pilz's, Near Ring Theory.

2.Preliminaries

Definition 2.1

A **Right Near-ring** is a set N with two binary operations '+' and '.' such that

a. (N, +) is a group (not necessarily abelian)
b. (N, .) is a semi group
c. (x + y).z = x.z + y.z for all x, y, z ∈ N (right distributive law)

Definition 2.2

i. N_0 = {n ∈ N / n0 = 0} is called the **zero-symmetric part** of N.
ii. N_c = {n ∈ N / n0=n}
= {n ∈ N / for all n' ∈ N, nn' = n} is called the **constant part** of N.

Definition 2.3

1. An element a ∈ N is called **idempotent** element if $a^2 = a$.
2. An element n ∈ N is **nilpotent** if $n^k = 0$ for some positive integer k.
3. An element a ∈ N is **central** element if ax = xa for all x ∈ N.
4. An element d ∈ N is **distributive** if d (n + n') = dn + dn' for all n, n' ∈ N

   N_d = {d ∈ N / d is distributive}

Definition 2.4

A right near ring N is said to satisfy **weak commutativity** if xyz = xzy for all x, y, z ∈ N.

Definition 2.5

A right near ring N is **pseudo commutative** if xyz = zyx for all x, y, z ∈ N.

Definition 2.6

A right near ring N is said to satisfy **quasi weak commutativity** if xyz = zyx for all x, y, z ∈ N.

Definition 2.7

A right near ring N is said to be **sub - commutative** if aN = Na for all a ∈ N.

Definition 2.8

A right near ring N is called a **β_1 – Near Ring** if xNy = Nxy for all x, y ∈ N

Definition 2.9

A right near ring N is called a **β_2 – Near Ring** if xNy = xyN for all x, y ∈ N

Definition 2.10

A near ring N is said to fulfil **IFP** provided that for all a, b in N, ab = 0 ⇒ anb = 0 for all n ∈ N

Definition 2.11

A near ring N has **(* , IFP)** if:

i. N has IFP
ii. For all a, b in N, ab = 0 ⇒ ba = 0

Definition 2.12

N has **Strong IFP** if every homomorphic image of N has IFP.

Definition 2.13

A near ring N is said to be **Boolean** if $a^2 = a$ for all a ∈ R

Definition 2.14

N is said to be **regular** if for each a ∈ N, there exists x ∈ N such that a = axa

3. R_1 – Near Ring

Definition 3.1

A near ring N is said to be **R_1 – Near Ring** if Nxy = xyN for all x, y ∈ N.

Theorem 3.2

If N is R_1 – near ring with identity, then N is sub-commutative.

Proof:

Since N is R_1 – near ring,

Nxy = xyN for every x, y ∈ N.

Put y = 1, ⇒ Nx = xN

⇒ N is sub-commutative

Theorem 3.3

Every sub-commutative near ring is R_1 – near ring if it is both weak & quasi weak commutative.

Proof:

First, to prove, Nxy ⊆ xyN

Let a ∈ Nxy ⇒ a = nxy = xny {N is sub-commutative} = xyn {N is weak commutative}

⇒ a ∈ xyN

⇒ Nxy ⊆ xyN

Now, to prove xyN ⊆ Nxy

Let b ∈ xyN ⇒ b = xyn' = xn'y {N is sub-commutative} = n'xy {quasi weak commutative}

⇒ b ∈ xyN

⇒ xyN ⊆ Nxy

Hence, Nxy = xyN

This completes the proof.

Theorem 3.4

Homomorphism preserves the quality of R_1 – near ring.

Proof:

Let f : N ⟶ N' be a near ring homomorphism.

For all x', y' $\in$ N' there exists x, y $\in$ N such that f(x) = x' and f(y) = y'

Also, for n' $\in$ N' there exists n $\in$ N such that f(n) = n'

Since, N is an R_1 – near ring, Nxy = xyN

n'x'y' = f(n)f(x)f(y) = f(nxy) = f(xyn) = f(x)f(y)f(n)

n'x'y' = x'y'n' $\in$ x'y'n'

$\Rightarrow$N'x'y'$\subseteq$ x'y'N'

Similarly, x'y'N' $\subseteq$ N'x'y'

$\Rightarrow$ N'x'y' = x'y'N'

$\Rightarrow$N' is a R_1 – near ring.

Hence the proof.

Theorem 3.5

In a R_1 – near ring, every β_1 – Near Ring is a β_2 – Near Ring and vice versa.

Proof:

First, $\beta_1 \Rightarrow \beta_2$

Taking R_1 – near ring N to be β_1 – Near Ring, we have,

xyN = Nxy

= xNy

= xyN

$\Rightarrow$ R_1 – near ring is also a β_2 – Near Ring

$\beta_2 \Rightarrow \beta_1$

Taking R_1 – near ring N to be β_1 – Near Ring, we have,

Nxy = xyN

= xNy

= Nxy

$\Rightarrow R_1$ – near ring is also a β_1 – Near Ring

This completes the proof.

Theorem 3.6

If every β_1 – Near Ring is a R_1 – near ring, then $x^2 = xNx$ for all $x \in N$

Proof:

For an R_1 – near ring being β_1 – Near Ring

xNx = Nxx = xxN = x2N = xNx

$\Rightarrow$ N is a P_2 – Near ring.

Theorem 3.7

If I is an ideal of an R_1 – near ring then N / I is also an R_1 – near ring.

Proof:

Let $f : N \longrightarrow N / I$ be a canonical near ring homomorphism given by, f(x) = I + x

Clearly, f is onto.

The proof follows from Theorem 3.4.

Theorem 3.8

Let N be a R_1 – near ring. If N is regular then for every $a \in N$, there exists $x \in N$ such that $xa^2 = a$

Proof:

Since N is regular, for every $a \in N$, there exists $x \in N$ such that a = axa

Since N is a R_1 – near ring, by above theorem 3.7, there exists $n \in N$ such that axa = xan

Thus, a = xan

$an = (axa)n = a(xan) = a.a = a^2$

$\Rightarrow a^2 = an$

Now, $xa^2 = xan \Rightarrow xa^2 = a$

Proposition 3.9

N has no non-zero nilpotent elements iff $a^2 = 0 \Rightarrow a = 0$ for all $a \in N$

Proposition 3.10

Let N be a R_1 – near ring. If N is regular then N has no non - zero nilpotent elements.

Proof:

Let $a \in N$.

Suppose $a^2 = 0$

By theorem 3.8, there exists $x \in N$ such that $a = xa2 = x0 = 0 \Rightarrow a = 0$.

By Proposition 3.9, N has no non – zero nilpotent elements.

Proposition 3.11

Anti homomorphism is preserved in a R_1 – near ring N whenever it is weak commutative.

Proof:

Let $f : N \longrightarrow N'$ be a near ring anti - homomorphism.

For all x’, y’ $\in$ N’ there exists x, y $\in$ N such that f(x) = x’ and f(y) = y’

Also, for n’ $\in$ N’ there exists n $\in$ N such that f(n) = n’

Since, N is an R_1 – near ring, Nxy = xyN

f(xyn) = f(n) f(y) f(x) = n’x’y’ = n’y’x’ = f(n)f(x)f(y)

Thus, anti – homomorphism is preserved.

Hence the proof.

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