A Bottle Neck Problem in Multiple Attribute Group Decision Making with the Application of Runge Kutta Merson Method

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ABSTRACT

In this paper, we have investigated Multiple Attribute Group Decision Making (MAGDM) Problems with Interval Valued Intuitionistic Fuzzy Sets (IVIFS). The decision maker weight information is completely unknown in the decision problem considered in this paper. The attribute weights are derived from Numerical Methods of Ordinary Differential Equation by using Runge Kutta Merson Method and it is applied in decision making problems. Correlation Coefficient is used for finding the rank of alternatives. Numerical illustration is given to show the effectiveness of the proposed method.

Keywords: Fuzzy Set, ODE, MAGDM Problem, Runge Kutta Merson Method.

I. INTRODUCTION

Multiple Attribute Group Decision Making (MAGDM) is the most well known branch of decision making. It is a branch of a general class of operations research models that deals with decision problems under the presence of a number of decision criteria. The MAGDM approach requires that the selection be made among decision alternatives described by their attributes. In many situations decision makers have imprecise/vague information about alternatives with respect to attributes. One of the methods which describe imprecise cases is the fuzzy set (FS) introduced by **Zadeh** [30] In classical fuzzy set theory there is no means to incorporate this hesitation regarding the degree of suitability to which each alternative satisfies the decision maker's requirement. To include the unknown degree in the membership function of fuzzy sets, **Atanassov** [1] introduced the concept of Intuitionistic Fuzzy Sets (IFSs). **Chen & Tan,** [2] was proposed multicriteria fuzzy decision making problems based on vague sets. **Park et al.** [9] also worked on the correlation coefficient of interval valued intuitionistic fuzzy sets and applied in MAGDM problems. **Robinson & Amirtharaj** [11-21]**, Robinson & Jeeva** [22-24] defined correlation coefficient for different higher order intuitionistic fuzzy sets and utilized in MAGDM problems. **Liu** [3] and **Liu & Guan** [5, 6] provided some new techniques for handling multiple attribute fuzzy decision making problems based on vague set theory. **Liu** [4] **Liu et al.,** [7]**, Wei** [25, 26]**, Wei et al.** [27,29] and **Wei & Zhao** [28] contributed novel approaches to the field of fuzzy decision making. In this work, numerical methods are proposed for determining weights of decision makers and used for MAGDM problems. **Jain et al.** [8] and **Rice** [10] discussed several numerical methods for scientific and engineering computation. In this work, Runge-Kutta Merson fourth order method is used to obtain the solution of Numerical methods which are utilized to derive the decision maker weights in MAGDM problems under interval valued intuitionistic fuzzy sets. Feasibility and effectiveness of the proposed method are illustrated using numerical examples.

DEFINITION 1: INTUITIONISTIC FUZZY SET (IFS)

Let a set X be fixed. An IFS Ã in X is an object having the form $\tilde{A} = \{< x,\ \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x) >,\ x \in X\}$, where the $\mu_{\tilde{A}}(x): X \to [0,\ 1]$ and $\gamma_{\tilde{A}}(x): E \to [0,\ 1]$ define the degree of membership and degree of non-membership respectively, of the element x∈X to the set Ã, which is a subset of X, for every element x ∈ X, $0 \le \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \le 1.$

DEFINITION 2: INTERVAL VALUED INTUITIONISTIC FUZZY SET (IVIFS)

Let a set X be fixed. An IFS Ã in X is an object having the form $\tilde{A} = \{< x,\ [\mu_{\tilde{A}L}(x), \mu_{\tilde{A}U}(x)], [\gamma_{\tilde{A}L}(x), \gamma_{\tilde{A}U}(x)] >,\ x \in X\}$, where the $[\mu_{\tilde{A}L}(x), \mu_{\tilde{A}L}(x)]: X \to [0,\ 1]$ and $[\gamma_{\tilde{A}L}(x), \gamma_{\tilde{A}U}(x)]: E \to [0,\ 1]$ define the degree of membership and degree of non-membership respectively, of the element x∈X to the set Ã, which is a subset of X, for every element x ∈ X, $0 \le [\mu_{\tilde{A}L}(x), \mu_{\tilde{A}U}(x)] + [\gamma_{\tilde{A}L}(x), \gamma_{\tilde{A}U}(x)] \le 1.$

DEFINITION 3: IVIFWAA OPERATOR

Let $\tilde{\alpha}_j, \left(j=1,2,...,n\right)$, be a collection of interval-valued intuitionistic fuzzy numbers. An Interval-valued Intuitionistic Fuzzy Weighted Arithmetic Averaging (IVIFWAA) operator of dimension n is a mapping IVIFWAA: $\Omega^n \to \Omega$, that has an associated vector $\omega=\left(\omega_1,\omega_2,....,\omega_n\right)^T$ such that $\omega_j \in \left[0,1\right]$ and $\sum_{j=1}^{n}\omega_j=1$.

THEOREM 1: Let $\tilde{\alpha}_j \left(j=1,2,.....,n\right)$ be a collection of interval-valued intuitionistic fuzzy numbers; then their aggregated value is also an interval-valued intuitionistic fuzzy number.

Let $\tilde{\alpha}_j, \left(j=1,2,.....,n\right)$, be a collection of interval-valued intuitionistic fuzzy numbers; then the aggregated value by using the IVIFWA operator is also an interval-valued intuitionistic fuzzy number and

$$IVIFWAA\left(\tilde{\alpha}_1,\tilde{\alpha}_2,.....,\tilde{\alpha}_n\right)=\sum_{j=1}^{n}\tilde{a}\omega_j=\left(\left[1-\prod_{j=1}^{n}\left(1-\mu^{-}_{\tilde{\alpha}_j}\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\mu^{+}_{\tilde{\alpha}_j}\right)^{\omega_j}\right],\left[\prod_{j=1}^{n}\left(\gamma^{-}_{\tilde{\alpha}_j}\right)^{\omega_j},\prod_{j=1}^{n}\left(\gamma^{+}_{\tilde{\alpha}_j}\right)^{\omega_j}\right]\right)$$

where $\omega=\left(\omega_1,\omega_2,....,\omega_n\right)^T$ is the weight vector of IVITzFWA operator with $\omega_j \in\left[0,1\right]$ and $\sum_{j=1}^{n}\omega_j=1$.

THEOREM 2: Let $\tilde{\alpha}_j, \left(j=1,2,.....,n\right)$, be a collection of interval-valued intuitionistic fuzzy numbers and $\omega=\left(\omega_1,\omega_2,....,\omega_n\right)^T$ be the weight vector of $\tilde{\alpha}_j, \left(j=1,2,.....,n\right)$, with $\omega_j\in\left[0,1\right]$ and $\sum_{j=1}^{n}\omega_j=1$; Then we prove that the IVIFWAA operator is (i) Idempotent, (ii) Bounded and (iii)Monotonic.

THEOREM 3: Let $\tilde{\alpha}_j, \left(j=1,2,.....,n\right)$, be a collection of interval-valued intuitionistic fuzzy numbers; then the aggregated value by using the IVIFOWA operators is also an interval-valued intuitionistic fuzzy number and

$$IVIFOWA_{\omega}\left(\tilde{\alpha}_1,\tilde{\alpha}_2,.....,\tilde{\alpha}_n\right)=\left(\left[1-\prod_{j=1}^{n}\left(1-\mu^{-}_{\tilde{\alpha}_{\sigma(1)}}\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\mu^{+}_{\tilde{\alpha}_{\sigma(1)}}\right)^{\omega_j}\right],\left[\prod_{j=1}^{n}\left(1-\gamma^{-}_{\tilde{\alpha}_{\sigma(1)}}\right)^{\omega_j},\prod_{j=1}^{n}\left(1-\gamma^{+}_{\tilde{\alpha}_{\sigma(1)}}\right)^{\omega_j}\right]\right),$$

where $\omega=\left(\omega_1,\omega_2,....,\omega_n\right)^T$ is the weight vector of the IVIFOWA operator with $\omega_j\in\left[0,1\right]$ and $\sum_{j=1}^{n}\omega_j=1$.

NOTE 1: The Commutative property holds for IVIFOWA operator and does not hold for IVIFWAA operator.

THEOREM 4: Let $\tilde{\alpha}_j, \left(j=1,2,.....,n\right)$, be a collection of interval-valued intuitionistic fuzzy numbers and $\omega=\left(\omega_1,\omega_2,.....,\omega_n\right)^T$ be the weight vector of IVIFOWA operator, with $\omega_j\in\left[0,1\right]$ and $\sum_{j=1}^{n}\omega_j=1$. Then we have the following:

1. If $\omega=\left(1,0,0,....,0\right)^T$, then $IVIFOWAA(\tilde{\alpha}_1,\tilde{\alpha}_2,....,\tilde{\alpha}_n)=\max_j\left(\tilde{\alpha}_j\right)$, .
2. If $\omega=\left(0,0,0,....,1\right)^T$, then $IVIFOWA_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2,....,\tilde{\alpha}_n)=\min_j\left(\tilde{\alpha}_j\right)$,
3. If $\omega_j=1$, $\omega_i=0$ and $i\neq j$, then $IVIFOWAA(\tilde{\alpha}_1,\tilde{\alpha}_2,....,\tilde{\alpha}_n)=\left(\tilde{\alpha}_{\sigma(j)}\right)$, where $\tilde{\alpha}_{\sigma(j)}$ is the jth largest of $\tilde{\alpha}_i, \left(i=1,2,...,n\right)$.

CORRELATION CO-EFFICIENT OF INTERVAL VALUED INTUITIONISTIC FUZZY SETS

Since an interval valued intuitionistic fuzzy number is characterized by interval valued intuitionistic fuzzy information and the fuzzy information, any previous method proposed in the literature for finding correlation coefficient for interval valued intuitionistic fuzzy sets will not be sufficient for working with the correlation coefficient of interval valued intuitionistic fuzzy sets. Thus there is a need to define a new type of correlation coefficient for interval valued intuitionistic fuzzy sets which will deal uniquely with both the interval valued intuitionistic fuzzy information and the fuzzy information, combining the two characterizations to derive a single correlation coefficient. The new method of finding the correlation coefficient for interval valued intuitionistic fuzzy numbers is as follows:

Let us consider two interval valued intuitionistic fuzzy sets as follows:
$A=([\mu_{AL}(x),\mu_{AU}(x)],[\gamma_{AL}(x),\gamma_{AU}(x)])$ and $B=([\mu_{BL}(x),\mu_{BU}(x)],[\gamma_{BL}(x),\gamma_{BU}(x)])$.

Consider the perfect interval-valued intuitionistic fuzzy numbers (called the Positive Ideal Solution-PIS and the Negative Ideal Solution-NIS): $\tilde{r}^{+}=([1,1],[0,0])$ and $\tilde{r}^{-}=([0,0],[1,1])$.

DEFINITION 4: For any two interval valued intuitionistic fuzzy sets $A=\left([\mu_{AL}(x),\mu_{AU}(x)],[\gamma_{AL}(x),\gamma_{AU}(x)]\right)$ and $B=\left(\left[\mu_{BL}(x),\mu_{BU}(x)\right],\left[\gamma_{BL}(x),\gamma_{BU}(x)\right]\right)$ the Interval Valued Intuitionistic Energy (IVIE) is defined as follows:

$$E_{IVIFS}(\text{A})=\frac{1}{2}\left(\mu_{AL}^{2}(\text{x})+\gamma_{AL}^{2}(\text{x})+\pi_{AL}^{2}(\text{x})+\mu_{AU}^{2}(\text{x})+\gamma_{AU}^{2}(\text{x})+\pi_{AU}^{2}(\text{x})\right),$$

$$E_{IVIFS}(\text{B})=\frac{1}{2}\left(\mu_{BL}^{2}(\text{x})+\gamma_{BL}^{2}(\text{x})+\pi_{BL}^{2}(\text{x})+\mu_{BU}^{2}(\text{x})+\gamma_{BU}^{2}(\text{x})+\pi_{BU}^{2}(\text{x})\right).$$

DEFINITION 5: For any two interval valued intuitionistic fuzzy sets $A=\left([\mu_{AL}(x),\mu_{AU}(x)],[\gamma_{AL}(x),\gamma_{AU}(x)]\right)$ and the correlation between the interval valued intuitionistic fuzzy sets A and B is defined as follows:

$$C_{IVIFS}(\text{A},\text{B})=\frac{1}{2}\begin{pmatrix}\mu_{AL}(\text{x})\mu_{BL}(\text{x})+\gamma_{AL}(\text{x})\gamma_{BL}(\text{x})+\pi_{AL}(\text{x})\pi_{BL}(\text{x})\\+\mu_{AU}(\text{x})\mu_{BU}(\text{x})+\gamma_{AU}(\text{x})\gamma_{BU}(\text{x})+\pi_{AU}(\text{x})\pi_{BU}(\text{x})\end{pmatrix}.$$

Then the correlation coefficient is given by,

$$K_{IVIFS}(A,B)=\frac{C_{IVIFS}(A,B)}{\sqrt{E_{IVIFS}(A).\text{E}_{IVIFS}(B)}},\quad 0\le K_{IVIFS}(A,B)\le 1.$$

NORMALIZATION OF WEIGHTING VECTOR FOR DECISION MAKING PROBLEM USING RUNGE-KUTTA MERSON METHOD

This method is the fourth order method even though five different $k's$ must be computed.

$$y_{j+1}=y_j+\frac{k_1+4k_4+k_5}{6}+O\left(h^5\right)$$

$$k_1=hf\left(x_j,y_j\right),\ k_2=hf\left(x_j+\frac{h}{3},y_j+\frac{k_1}{3}\right),\ k_3=hf\left(x_j+\frac{h}{3},y_j+\frac{k_1}{6}+\frac{k_2}{6}\right),\ k_4=hf\left(x_j+\frac{h}{2},y_j+\frac{k_1}{8}+\frac{3k_3}{8}\right)$$

$$k_5=hf\left(x_j+h,y_j+\frac{k_1}{2}-\frac{3k_3}{2}+2k_4\right),\ \text{Error } E=\frac{1}{30}\left(2k_1-9k_3+8k_4-k_5\right)$$

EXAMPLE

Apply Runge-Kutta-Merson method solving the given equation

$\frac{dy}{dx}=-y$ and $y\left(0\right)=1$

soln:

Given $\frac{dy}{dx}=-y$.

Here, $x_0=0, y_0=1.$

Let $h=0.1$

$k_1=(0.1)\,f\,(0,0.1)=-0.1$

$k_2=(0.1)\,f\left(0+\frac{0.1}{3},1+\frac{(-0.1)}{3}\right)=-0.096666666$

$k_3=(0.1)\,f\,(0.3333,0.967222222)=-0.096722222$

$k_4=(0.1)\,f\,(0.05,0.951229166)=-0.095122916$

$k_5=(0.1)\,f\,(0.1,0.904837501)=-0.09048375$

The approximate value of $y_1=y(0.1)=0.934998681.$

Similarly, we obtain:

$y_2=0.846021804;\ y_3=0.765512195;\ y_4=0.692664088;\ y_5=0.626748394;\ y_6=0.567105406;$
$y_7=0.514166668;\ y_8=0.465237247;\ y_9=0.420964076;\ y_{10}=0.380904053.$

$\therefore$ Normalization of $y=\left(\frac{y_1}{y}+\frac{y_2}{y}+\frac{y_3}{y}+\frac{y_4}{y}+\frac{y_5}{y}\right)$

Normalization of $y\approx 1$

ALGORITHM FOR MAGDM WITH CORRELATION COEFFICIENT

Step-1: Utilize the IVIWAA for the decision matrix $\tilde{R}_k$, to derive the individual overall preference Interval Valued Intuitionistic Fuzzy Set (IVIFS) values,

$$\tilde{r}_i^{(k)}=IVIFWAA(\tilde{r}_{i1}^{\,k},\tilde{r}_{i2}^{\,k},....\tilde{r}_{in}^{\,k}), i=1,2,\ldots,m,\ k=1,2,\ldots,t.$$

Step-2: To calculate the correlation coefficient between collective overall values $\tilde{r}_i=([\mu_L,\mu_U],[\gamma_L,\gamma_U])$ and the interval valued intuitionistic fuzzy positive ideal solution $\tilde{r}^+=([\mu_L,\mu_U],[\gamma_L,\gamma_U])=([1,1],[0,0])$.

$$K_{IVIFS}(\tilde{r}_i,\tilde{r}^+)=C_{IVIFS}(\tilde{r}_i,\tilde{r}^+)\Big/\sqrt{E_{IVIFS}(\tilde{r}_i).E_{IVIFS}(\tilde{r}^+)}.$$

Step-3: Rank all the alternatives A_i *(i=1,2,….,m)* and select the best one in accordance with the correlation coefficient obtained in step 3.

NUMERICAL ILLUSTRATION

A city is planning to build a municipal library. One of the problems facing the city development commissioner is to determine what kind of air-conditioning systems should be installed in the library. The contractor offers five feasible alternatives, which might be adapted to the physical structure of the library. The offered air-conditioning system must take a decision according to the following five attributes: C1-Performance; C2-Maintainability; C3-Flexibility; C4-Cost; C5-Safety. The five possible alternatives $A_i\,(i=1,2,3,4,5)$ are to be evaluated using interval valued intuitionistic fuzzy numbers by the decision makers under the above five attributes runge kutta merson method weighting vector is $\omega=(0.241851702,0.218836472,0.198011431,0.179168155,0.162118053)^T$ and construct, respectively, the decision matrices as listed in the following matrices $R=\left(r_{2ij}^{(k)}\right)5*5$ as follows:

$$R=\begin{bmatrix} ([0.5,0.4],\ [0.4,0.3]) & ([0.6,0.3],\ [0.3,0.6]) & ([0.3,0.6],\ [0.6,0.3]) & ([0.2,0.7],\ [0.7,0.1]) & ([0.1,0.4],\ [0.7,0.1]) \\ ([0.6,0.2],\ [0.3,0.6]) & ([0.7,0.1],\ [0.3,0.6]) & ([0.7,0.2],\ [0.2,0.7]) & ([0.1,0.9],\ [0.8,0.1]) & ([0.7,0.1],\ [0.3,0.5]) \\ ([0.1,0.2],\ [0.6,0.3]) & ([0.4,0.5],\ [0.3,0.5]) & ([0.1,0.7],\ [0.6,0.2]) & ([0.2,0.6],\ [0.6,0.3]) & ([0.3,0.7],\ [0.6,0.4]) \\ ([0.3,0.4],\ [0.8,0.1]) & ([0.7,0.2],\ [0.6,0.4]) & ([0.4,0.3],\ [0.4,0.3]) & ([0.1,0.6],\ [0.6,0.2]) & ([0.1,0.8],\ [0.8,0.1]) \\ ([0.6,0.3],\ [0.2,0.7]) & ([0.7,0.2],\ [0.4,0.4]) & [0.5,0.5],\ [0.4,0.4] & ([0.1,0.7],\ [0.7,0.2]) & ([0.3,0.6],\ [0.6,0.2]) \end{bmatrix}$$

By using the algorithm we obtain:

$K_1 = 0.6593; K_2 = 0.5659; K_3 = 0.5555; K_4 = 0.6163; K_5 = 0.7270$

Rank all the alternatives $A_i\ (i=1,2,3,4,5)$.

$A_5 > A_1 > A_4 > A_2 > A_3$. Hence, Best alternative is A_5.

CONCLUSION

The MAGDM problems in which the attribute weights and the expert weights take the form of real numbers, attribute values take the form of interval valued intuitionistic fuzzy numbers were investigated in this paper. Some desirable properties were studied and applied the IVIFWAA operator to MAGDM problems with interval valued intuitionistic fuzzy information. Correlation coefficient was also proposed for ranking the alternatives. Finally, an illustrative example was given to show the effectiveness of proposed method.

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