Efficient Multiple Attribute Group Decision Making Problems using Sumudu Decomposition Method in Partial Differential Equation

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Abstract: In this paper, the unknown weights of the decision maker can be determined using the exact solution of partial differential equation solved through sumudu decomposition method. For the decision making Intuitionistic Triangular Fuzzy Weighted Geometric (ITrFWG) operator and Intuitionistic Triangular Fuzzy Hybrid Geometric (ITrFHG) operator are utilized. The distance measure is used to identify the desirable alternative from the available alternatives. A comparisons with existing methods is also made and the choice of the best alternatives.

Key words: Intuitionistic Fuzzy Set, ITrFWG and ITrFHG operator, Sumudu Decomposition Method, Partial Differential Equation.

I.INTRODUCTION

Decision Making is very import role for human's life but some situations is critical and undefined. This situation is great deal of fuzziness, fuzzy set was proposed in [27] and an intuitionistic fuzzy set was proposed in [2]. In [5, 26], a novel proposed intuitionistic correlation coefficient fuzzy sets. Decision Making Problems are solved by using different aggregation operators like arithmetic and geometric operators are proposed in [3, 24-25]. In [11, 12], a novel discussed Multiple attribute group decision analysis for intuitionistic triangular and trapezoidal fuzzy sets. A new method for determining the unknown decision maker weights using singularly perturbation problems are proposed in [13-15, 22-23]. In [9], a novel proposed a new ratio ranking method under triangular intuitionistic fuzzy number and its applications. In [6-8, 16-21], a novel finding the unknown decision maker weights using entropy sumudu transform in various intuitionistic fuzzy environment. In [1], a novel discussed the solution Burgers equation with the help of Sumudu Decomposition Method. A linear and Non-linear Klein-Gordon equations are solved through Sumudu Decomposition Method is proposed in [10]. Sumudu transform is very effective for solving differential and integral equations. In [4], a novel investigated the sumudu transform fundamental properties and its applications. In this work, a newly find the unknown decision maker weights are derived from partial differential equation are solved through Sumudu Decomposition Method and it was applied in decision making problem. Numerical Illustration is given to show the effectiveness of the proposed approach. A comparison made with given existing methods.

II. PRELIMINARIES

In this section, some basic definitions and geometric aggregation operators of Intuitionistic Triangular Fuzzy Sets are presented.

Definition: 1 Intuitionistic Fuzzy Set

Let a set X be fixed. An IFS Ã in X is an object having the form $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x) >, x \in X\}$, where the $\mu_{\tilde{A}}(x): X \to [0, 1]$ and $\gamma_{\tilde{A}}(x): E \to [0, 1]$ define the degree of membership and degree of non-membership respectively, of the element x∈X to the set Ã, which is a subset of X, for every element x ∈ X, $0 \le \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \le 1$.

Definition: 2 Intuitionistic Fuzzy Number

1. An IFN Ã is defined as follows:
2. An intuitionistic fuzzy sub set of the real line.
3. Normal i.e there is any $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$ (so $\gamma_{\tilde{A}}(x_0) = 0$).
4. Convex for the membership function $\mu_{\tilde{A}}(x)$
   i.e $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, $x_{1,}x_2 \in R$, $\lambda \in [0,1]$.
5. Concave for the non- membership function $\gamma_{\tilde{A}}(x)$
   i.e $\gamma_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max\{\gamma_{\tilde{A}}(x_1), \gamma_{\tilde{A}}(x_2)\}, x_{1,}x_2 \in R$, $\lambda \in [0,1]$.

Definition: 3 Triangular Fuzzy Number (TrFN) $A = (a,b,c)$ is called a triangular fuzzy number, if the membership function $\mu_A : R \to [0,1]$ is expressed as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{x-c}{b-c} & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

where $x \in R$, $0 \leq a \leq b \leq c \leq 1$.

III. SOME GEOMETRIC AGGREGATION OPERATORS WITH INTUITIONISTIC TRIANGULAR FUZZY NUMBERS

Defintion: 4 Let $\tilde{\alpha}_j\,(j = 1,2,...,n)$ be collection of intuitionistic triangular fuzzy numbers, and let $\text{ITrFWG}:\Omega^n \to \Omega$. Then $\text{ITrFWG}_\omega(\tilde{\alpha}_1, \tilde{\alpha}_1, ..., \tilde{\alpha}_n) = \tilde{\alpha}_1^{\omega_1} \otimes \tilde{\alpha}_1^{\omega_1} .... \otimes \tilde{\alpha}_1^{\omega_1}$, is called the Intuitionistic Triangular Fuzzy Weighted Geometric (ITrFWG) operator.

Theorem: 1 Let $\tilde{\alpha}_j\,(j = 1,2,...,n)$ be a collection of intuitionistic triangular fuzzy numbers. Then the aggregated value by using the ITrFWG operator is also an intuitionistic triangular fuzzy number and

$$\text{ITrFWG}_\omega(\tilde{\alpha}_1, \tilde{\alpha}_1, ..., \tilde{\alpha}_n) = \prod_{j=1}^{n} \tilde{\alpha}_j^{\omega_j} = \left(\left[\prod_{j=1}^{n} \alpha_j^{\omega_j}, \prod_{j=1}^{n} b_j^{\omega_j}, \prod_{j=1}^{n} c_j^{\omega_j} \right]; \prod_{j=1}^{n} \mu_{\tilde{\alpha}_j}^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - \gamma_{\tilde{\alpha}_j}\right)^{\omega_j} \right),$$

where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weight vector of $\tilde{\alpha}_j\,(j = 1,2,...,n)$, with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Definition: 5 Let $\tilde{\alpha}_j\,(j = 1,2,...,n)$ be a collection of intuitionistic triangular fuzzy numbers. An Intuitionistic Triangular Fuzzy Ordered Weighted Geometric (ITrFOWG) operator of dimension n is a mapping $\text{ITrFOWG}:\Omega^n \to \Omega$ that has an associated vector $w = (w_1, w_2, ..., w_n)^T$ such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, Furthermore

$$ITrFOWG_w(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \tilde{\alpha}_{\sigma(1)}^{w_1} \otimes \tilde{\alpha}_{\sigma(2)}^{w_2} \otimes ... \tilde{\alpha}_{\sigma(n)}^{w_n},$$

where $(\sigma(1), \sigma(2), ... \sigma(n))$ is a permutation of (1,2,…,n) such that $\tilde{\alpha}_{\sigma(j-1)} \geq \tilde{\alpha}_{\sigma(j)}$ for all j.

$$\text{ITrFOWG}_\omega(\tilde{\alpha}_1, \tilde{\alpha}_1, ..., \tilde{\alpha}_n) = \prod_{j=1}^{n} \tilde{\alpha}_{\sigma(j)}^{\omega_j} = \left(\left[\prod_{j=1}^{n} \alpha_{\sigma(j)}^{\omega_j}, \prod_{j=1}^{n} b_{\sigma(j)}^{\omega_j}, \prod_{j=1}^{n} c_{\sigma(j)}^{\omega_j} \right]; \prod_{j=1}^{n} \mu_{\tilde{\alpha}_{\sigma(j)}}^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - \gamma_{\tilde{\alpha}_{\sigma(j)}}\right)^{\omega_j} \right).$$

Definition: 6 Let $\tilde{\alpha}_j\left(j=1,2,...,n\right)$ be a collection of intuitionistic triangular fuzzy numbers. An Intuitionistic Triangular Fuzzy Hybrid Geometric (ITrFHG) operator of dimention n is a mapping $\text{ITrFHG}:\Omega^n \to \Omega$ that has an associated vector $\omega=\left(\omega_1,\omega_2,...,\omega_n\right)^T$ such that $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j=1.$

$$\text{ITrFHG}_{\omega,w}\left(\tilde{\alpha}_1,\tilde{\alpha}_1,...,\tilde{\alpha}_n\right)=\dddot{\alpha}_{\sigma(1)}^{\omega_j}\otimes\dddot{\alpha}_{\sigma(2)}^{\omega_j}...\otimes\dddot{\alpha}_{\sigma(n)}^{\omega_j},$$

where $\dddot{\alpha}_{\sigma(j)}^{\omega_j}$ is the jth largest of the weighted intuitionistic triangular fuzzy numbers $\ddot{\alpha}_j^{\omega_j}\left(\dddot{\alpha}_j^{\omega_j}=\dddot{\alpha}_j^{n\omega_j},\ j=1,2,...,n\right)$. $w=\left(w_1,w_2,...,w_n\right)^T$ is the weight vector of $\tilde{\alpha}_j$ with $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1,$ and n is the balancing coefficient, in such case, if the vector $w=\left(w_1,w_2,...,w_n\right)^T$ approaches $\left(1/n,1/n,...,1/n\right)^T$, then the vector $\left(\tilde{\alpha}_1^{n\omega_1},\tilde{\alpha}_1^{n\omega_2},...,\tilde{\alpha}_1^{n\omega_n}\right)^T$ approaches $\left(\tilde{\alpha}_1,\tilde{\alpha}_1,...,\tilde{\alpha}_n\right)^T$.

$$\text{ITrFHG}_{w,\omega}\left(\tilde{\alpha}_1,\tilde{\alpha}_1,...,\tilde{\alpha}_n\right)=\prod_{j=1}^{n}\tilde{\alpha}_{\sigma(j)}^{\omega_j}=\left(\left[\prod_{j=1}^{n}\ddot{\alpha}_{\sigma(j)}^{\omega_j},\prod_{j=1}^{n}\dddot{b}_{\sigma(j)}^{\omega_j},\prod_{j=1}^{n}\ddot{c}_{\sigma(j)}^{\omega_j}\right];\prod_{j=1}^{n}\ddot{\mu}_{\tilde{\alpha}_{\sigma(j)}}^{\omega_j},1-\prod_{j=1}^{n}\left(1-\ddot{\gamma}_{\tilde{\alpha}_{\sigma(j)}}\right)^{\omega_j}\right).$$

IV. DETERMINING EXPERTS WEIGHTS FOR MAGDM PROBLEMS USING SUMUDU DECOMPOSITION METHOD

Definition: 7 (Sumudu Transform) Consider functions in the set A, defined by

$A=\left\{f(t)\,|\,\exists M,\tau_1, and/or\ \tau_2>0,\ \text{suchthat}\ |f(t)|<Me^{|t|/\tau_j},\ \text{if}\ t\in(-1)^j\times[0,\infty)\right\}$. For a given function in the set A, the constant M must be finite, while τ_1 and τ_2 need not simultaneously exist, and each may be infinite. Insteed of being used as a power to the exponential as in the case of the Laplace transform, the variable u in the Sumudu transform is used to factor the variable t in the argument of the function f. Specifically for $f(t)$ in A, the sumudu transform is defined by

$$G(u)=S\left[f(t)\right]=\begin{cases}\int_0^{\infty}f(ut)e^{-t}dt, & 0\le u\le\tau_2,\\ \int_0^{\infty}f(ut)e^{-t}dt, & -\tau_1\le u\le 0.\end{cases}$$

Sumudu Decomposition Method

To illustrate the basic idea of this method, We consider a general nonlinear non-homogeneous partial differential equation:

$$DU(x,t)+RU(x,t)+NU(x,t)=g(x,t) \quad (1)$$

$$U(x,0)=h(x),\ U_t(x,0)=f(x),$$

where D is the second order linear differential operator $D=\frac{\partial^2}{\partial t^2}$, R is the linear differential operator of less order than D, N represents the general nonlinear differential operator and $g(x,t)$ is the source term. Taking the Sumudu transform on both sides of equation (1), we get:

$$S[DU(x,t)]+S[RU(x,t)]+S[NU(x,t)]=S[g(x,t)]. \quad (2)$$

Using the differentiation property of the Sumudu transform and above initial conditions, we have

$$S[DU(x,t)]=Su^2[g(x,t)]+h(x)+uf(x)-Su^2[RU(x,t)+NU(x,t)]. \quad (3)$$

Now, applying the inverse Sumudu transform on both sides of equation (3) we obtain:

$$U(x,t)=G(x,t)-S^{-1}\left[Su^2[RU(x,t)+NU(x,t)]\right], \quad (4)$$

where, $G(x,t)$ represents the term arising from the source term and the prescribed initial conditions. The second step in Sumudu Decomposition Method is that we represent solution as an infinite series given below

$$U(x,t)=\sum_{n=0}^{\infty}U_n(x,t), \tag{5}$$

and the nonlinear term can be decomposed as : $NU(x,t)=\sum_{n=0}^{\infty}A_n,$ (6)

where A_n are Adomain polynomials of $U_0,U_1,U_2,\ldots,U_n$ and it can be calculated by formula

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{0}^{\infty}\lambda^i U_i\right)\right]_{\lambda=0}, n=0,1,2,\ldots .$$

Using Equation (5) and Equatin (6) in Equation (4), we get

$$\sum_{n=0}^{\infty}U(x,t)=G(x,t)-S^{-1}\left[Su^2\left(R\sum_{n=0}^{\infty}U_n(x,t)+\sum_{n=0}^{\infty}A_n\right)\right]. \tag{7}$$

On comparing both sides of equation (7), we get

$$U_0(x,t)=G(x,t),$$
$$U_1(x,t)=-S^{-1}\left[Su^2\left(RU_0(x,t)+A_0\right)\right],$$
$$U_2(x,t)=-S^{-1}\left[Su^2\left(RU_1(x,t)+A_1\right)\right],$$
$$U_3(x,t)=-S^{-1}\left[Su^2\left(RU_2(x,t)+A_2\right)\right].$$

….

In general the recursive relation is given by

$$U_0(x,t)=G(x,t),$$
$$U_{n+1}(x,t)=-S^{-1}\left[Su^2\left(RU_n(x,t)+A_n\right)\right] \quad n\geq 0.$$

Problem proposed by Decision Maker 1:

The decision maker represents weighting in the form of partial differential equation $U_{tt}(x,t)-U_{xx}(x,t)+U(x,t)=0,$ with the initial conditions $U(x,0)=0,\ U(x,t)=0.$

By taking sumudu transform then we have $S\left[U(x,t)\right]=ux+u^2S[U_{xx}(x,t)-U_{tt}(x,t)].$

By applying inverse Sumudu Transform in above equation then we obtain

$$U(x,t)=ut+S^{-1}\left[u^2S[U_{xx}(x,t)-U_{tt}(x,t)]\right].$$

Using the decomposition series for the linear term $U(x,t)$ and the series of Adomain polynomials for the nonlinear term U_{tt} gives $\sum_{n=0}^{\infty}U_{n+1}(x,t)=ut+S^{-1}\left[u^2S\left[\sum_{n=0}^{\infty}(U_n)_{xx}(x,t)-\sum_{n=0}^{\infty}(U_n)_{tt}(x,t)\right]\right].$

This equation don't have the nonlinear term, then $A_n=0.$

From Adomain Polynomials,

$$U_0(x,t)=G(x,t)=xt,$$
$$U_1(x,t)=-S^{-1}\left[Su^2[(U_0)_{xx}(x,t)+(U_0)(x,t)]\right]=-\frac{xt^3}{6},$$
$$U_2(x,t)=-S^{-1}\left[Su^2[(U_1)_{xx}(x,t)+(U_1)(x,t)]\right]=\frac{xt^5}{120},$$
$$U_3(x,t)=-S^{-1}\left[Su^2[(U_2)_{xx}(x,t)+(U_2)(x,t)]\right]=-\frac{xt^7}{5040},$$

….

which is closed form gives exact solution

$$U(x,t)=\sum_{i=0}^{\infty}U_i(x,t)=x\left(t-\frac{t^3}{3!}+\frac{t^5}{5!}-\cdots\right)=x\sin t.$$

Table 1. Exact Solution of $U(x,t)=x\sin t$.

x	t	$U(x,t)$	$w_i=\frac{u(x,t)}{\sum u(x,t)}$
0.2	0.2	0.0397338661590122	0.0358553382878558
0.4	0.4	0.155767336923460	0.140562474772476
0.6	0.6	0.338785484037021	0.305715736005901
0.8	0.8	0.573884872719618	0.517866450933768

Problem proposed by Decision Maker 2:

The decision maker represents weighting in the form of partial differential equation $U_t=U_{xx}-UU_x$ and the initial condition $U(x,0)=x$.

Solution:

Given Equation $U_t=U_{xx}-UU_x$.

Taking the sumudu transform on both sides of equation and making use of initial condition to obtain

$$S[U(x,t)]=x+US[U_{xx}]-US[UU_x].$$

Applying the inverse sumudu transform implies that

$$U(x,t)=x+S^{-1}\left[US[U_{xx}]\right]-S^{-1}\left[US[UU_x]\right].$$

Using the decomposition series for the linear term $U(x,t)$ and the series of the Adomain polynomials for the nonlinear term UU_x gives $\sum_{n=0}^{\infty}U_n(x,t)=x+S^{-1}\left[US\left[\sum_{n=0}^{\infty}(U_n)_{xx}(x,t)\right]\right]-S^{-1}\left[US\left[\sum_{n=0}^{\infty}A_n\right]\right]$.

This leads to the recursive relation $U_0(x,t)=x$,

$$U_{k+1}(x,t)=S^{-1}\left[U\left(S\left(U_x\right)_{xx}\right)\right]-S^{-1}\left(US(A_k)\right),\ K\geq 0.$$

By using Adomian Polynomials,

$U_1(x,t)=S^{-1}\left[US(U_0)_{xx}\right]-S^{-1}\left[US(A_0)\right]=-xt$,

$U_2(x,t)=S^{-1}\left[US(U_1)_{xx}\right]-S^{-1}\left[US(A_1)\right]=xt^2$,

$U_3(x,t)=S^{-1}\left[US(U_2)_{xx}\right]-S^{-1}\left[US(A_2)\right]=-xt^3$, And so on.

The solution of the series is given by $U(x,t)=x(1-t+t^2-t^3+\ldots)$.

The closed form solution is given by $U(x,t)=\frac{x}{1+t}$.

Two sets of weights can be calculated from the problem proposed by decision maker 2.

Table 2. Exact Solution of $U(x,t)=x/1+t$.

x	t	$U(x,t)$	$w_i=\frac{u(x,t)}{\sum u(x,t)}$
0.1	0.1	0.0909090909090909	0.0613259203099992
0.2	0.2	0.166666666666667	0.112430853901665
0.3	0.3	0.230769230769231	0.155673490017690
0.4	0.4	0.285714285714286	0.192738606688569
0.5	0.5	0.333333333333333	0.224861707803330
0.6	0.6	0.375000000000000	0.252969421278746

V. AN APPROACH TO MAGDM PROBLEMS WITH INTUITIONISTIC TRIANGULAR FUZZY INFORMATION

Step 1: Utilize the decision information given in the intuitionistic triangular fuzzy decision matrix $\tilde{R}_k$, and the ITrFWG operator
$\tilde{r}_i^{(k)} = ([a_i^{(k)}, b_i^{(k)}, c_i^{(k)}]; \mu_i, \gamma_i) = ITrFWG_\omega(\tilde{r}_{i1}{}^k, \tilde{r}_{i2}{}^k, ...\tilde{r}_{in}{}^k)$, $i=1,2,...,m,\ k=1,2,...,t,$ to derive the individual overall interval valued intuitionistic triangular fuzzy numbers $\tilde{r}_i^{(k)}$ of the alternative A_i.

Step 2: Utilize the ITrFHG operator to derive the collective overall intuitionistic triangular fuzzy values $\tilde{r}_i, (i=1,2,....m),$ of the alternative A_i :

$$\tilde{r}_i = ([a_i, b_i, c_i]; \mu_i, \gamma_i) = ITrFHG_{v\ w}(\tilde{r}_i^1, \tilde{r}_i^2, ...\tilde{r}_i^t), \quad i=1,2,....,m.$$

where $v = (v_1, v_2, ..., v_n)$ is the weighting vector of decision makers, with $v_j \in [0,1],\ \sum_{j=1}^{k} v_k = 1;\ \ w = (w_1, w_2, ..., w_n)$ is the associated weighting vector of the ITrFHG operator, with $w_j \in [0,1],\ \sum_{j=1}^{n} w_j = 1.$

Step 3: Calculate the correlation coefficient between collective overall values
$r_i = ([a_i, b_i, c_i]; \mu_i, \gamma_i)$, and the triangular intuitionistic fuzzy positive ideal value: $\rho_i = (\rho_{cr-TR}(\tilde{r}_i, \tilde{r}^+)).$

Step 4: Calculate the distance measure between the two different correlation coefficient in the tuple which is given by:

$$d_i(\rho_{Cr-TR}(\tilde{r}_i, \tilde{r}^+), \rho_{ZL_i}(\tilde{r}_i, \tilde{r}^+)) = \left| \rho_{Cr-Tr}(\tilde{r}_i, \tilde{r}^+) - \rho_{ZL_i}(\tilde{r}_i, \tilde{r}^+) \right|,$$

where,

$$\rho_{Cr-TR}(A,B) = \frac{(b_i + a_i)(b^+ + a^+) + (c_i + b_i)(c^+ + b^+)}{\sqrt{(b_i + a_i)(b^+ + a^+) + (c_i + b_i)(c^+ + b^+)}},\ \rho_{ZL}(A,B) = Cl_{ZL}(a,b) \Big/ \sqrt{C_{ZL}(A,A).C_{ZL}(B,B)}.$$

Step5: Rank alternative $A_i (i=1,2,3......., m)$ and select one in accordance with $d_i(\rho_{Cr-TR}(\tilde{r}_i, \tilde{r}^+), \rho_{ZL_i}(\tilde{r}_i, \tilde{r}^+))$, i=1,2,3,....,m. The smaller $d_i(\rho_{Cr-TR}(\tilde{r}_i, \tilde{r}^+), \rho_{ZL_i}(\tilde{r}_i, \tilde{r}^+))$, the better alternatives A_i , when positive ideal value is taken.

V1. NUMERICAL ILLUSTRATION

Suppose an engineering investment company wants to invest a sum of money in the best option. There is a panel with four possible alternatives in which to invest the money:

1. A1 A car company;
2. A2 A food company;
3. A3 A computer company;
4. A4 An arms company.

The engineering investment company must make a decision according to the following five attributes:

1. C1 Risk analysis;
2. C2 Growth analysis;
3. C3 Social–political impact analysis;
4. C4 Environmental impact analysis;
5. C5 Technological analysis.

The five possible alternatives $A_i\,(i=1,2,3,4,5)$ are to be evaluated using intuitionistic triangular fuzzy numbers by the three decision makers whose weighting vector is obtained by normalizing the solution of partial differential equation proposed by decision maker 2 are $\gamma = (0.0613259203099992,\ 0.155673490017690,\ 0.224861707803330)^T$

$w=\left(0.112430853901665,\ 0.192738606688569,\ 0.252969421278746\right)^{T}$ under the above four attributes whose weighting vector is obtained by normalizing the solution of partial differentialequation proposed by decision maker 1 is

$w=\left(0.0358553382878558,\ 0.140562474772476,\ 0.305715736005901,\ 0.517866450933768\right)^{T}$ and

construct, respectively, the decision matrices as listed in the following matrices $R=\left(r_{2ij}^{(k)}\right)5*4\left(k=1,2,3\right)$ as follows:

$$R_1=\begin{pmatrix} ([0.5,0.6,0.7];0.5,0.4) & ([0.1,0.2,0.3];0.6,0.3) & ([0.5,\ 0.6,\ 0.8];0.3,\ 0.6) & ([0.4,\ 0.5,\ 0.6];0.2,\ 0.7) \\ ([0.6,0.7,0.8];0.7,0.3) & ([0.5,0.6,0.7];0.7,0.2) & ([0.4,\ 0.5,\ 0.7];0.7,\ 0.2) & ([0.5,\ 0.6,\ 0.7];0.4,\ 0.5) \\ ([0.1,0.2,0.4];0.6,0.4) & ([0.2,0.3,0.5];0.5,0.4) & ([0.5,\ 0.6,\ 0.7];0.5,\ 0.3) & ([0.3,\ 0.5,\ 0.7];0.2,\ 0.3) \\ ([0.3,0.4,0.5];0.8,0.1) & ([0.1,0.3,0.4];0.6,0.3) & ([0.1,\ 0.3,\ 0.5];0.3,\ 0.4) & ([0.6,\ 0.7,\ 0.8];0.2,\ 0.6) \\ ([0.2,0.3,0.4];0.6,0.2) & ([0.3,0.4,0.5];0.4,0.3) & ([0.2,\ 0.3,\ 0.4];0.7,\ 0.1) & ([0.5,\ 0.6,\ 0.7];0.1,\ 0.3) \end{pmatrix}$$

$$R_2=\begin{pmatrix} ([0.4,\ 0.5,\ 0.6];0.4,\ 0.3) & ([0.1,\ 0.2,\ 0.3];0.5,\ 0.2) & ([0.4,\ 0.5,\ 0.7];0.2,\ 0.5) & ([0.3,\ 0.4,\ 0.5];0.1,\ 0.6) \\ ([0.5,\ 0.6,\ 0.7];0.6,\ 0.2) & ([0.4,\ 0.5,\ 0.6];0.6,\ 0.1) & ([0.3,\ 0.4,\ 0.6];0.6,\ 0.1) & ([0.4,\ 0.5,\ 0.6];0.3,\ 0.4) \\ ([0.1,\ 0.2,\ 0.3];0.5,\ 0.3) & ([0.1,\ 0.2,\ 0.4];0.4,\ 0.3) & ([0.4,\ 0.5,\ 0.6];0.4,\ 0.2) & ([0.2,\ 0.4,\ 0.6];0.5,\ 0.2) \\ ([0.2,\ 0.3,\ 0.4];0.7,\ 0.1) & ([0.1,\ 0.2,\ 0.3];0.5,\ 0.2) & ([0.1,\ 0.2,\ 0.4];0.2,\ 0.3) & ([0.5,\ 0.6,\ 0.7];0.1,\ 0.5) \\ ([0.1,\ 0.2,\ 0.3];0.5,\ 0.1) & ([0.2,\ 0.3,\ 0.4];0.3,\ 0.2) & ([0.1,\ 0.2,\ 0.3];0.6,\ 0.2) & ([0.4,\ 0.5,\ 0.6];0.4,\ 0.2) \end{pmatrix}$$

$$R_3=\begin{pmatrix} ([0.6,\ 0.7,\ 0.8];0.4,\ 0.5) & ([0.2,\ 0.3,\ 0.4];0.5,\ 0.4) & ([0.6,\ 0.7,\ 0.9];0.2,\ 0.7) & ([0.5,\ 0.6,\ 0.7];0.1,\ 0.8) \\ ([0.7,\ 0.8,\ 0.9];0.6,\ 0.4) & ([0.6,\ 0.7,\ 0.8];0.6,\ 0.3) & ([0.5,\ 0.6,\ 0.8];0.6,\ 0.3) & ([0.6,\ 0.7,\ 0.8];0.3,\ 0.6) \\ ([0.2,\ 0.3,\ 0.5];0.5,\ 0.5) & ([0.3,\ 0.4,\ 0.6];0.4,\ 0.5) & ([0.6,\ 0.7,\ 0.8];0.4,\ 0.4) & ([0.4,\ 0.6,\ 0.8];0.5,\ 0.4) \\ ([0.4,\ 0.5,\ 0.6];0.7,\ 0.2) & ([0.2,\ 0.4,\ 0.5];0.5,\ 0.4) & ([0.2,\ 0.4,\ 0.6];0.2,\ 0.5) & ([0.7,\ 0.8,\ 0.9];0.1,\ 0.7) \\ ([0.3,\ 0.4,\ 0.5,];0.5,\ 0.3) & ([0.4,\ 0.5,\ 0.6];0.3,\ 0.4) & ([0.3,\ 0.4,\ 0.5];0.6,\ 0.2) & ([0.6,\ 0.7,\ 0.8];0.4,\ 0.4) \end{pmatrix}$$

Using the above decision making algorithm,

$d(\rho_{Cr-TR}(\tilde{r}_1, \tilde{r}^+), \rho_{ZL_1}) = 0.131726560524531;$

$d(\rho_{Cr-TR}(\tilde{r}_2, \tilde{r}^+), \rho_{ZL_1}) = 0.146991934208555;$

$d(\rho_{Cr-TR}(\tilde{r}_3, \tilde{r}^+), \rho_{ZL_1}) = 0.268506119264333;$

$d(\rho_{Cr-TR}(\tilde{r}_4, \tilde{r}^+), \rho_{ZL_1}) = 0.211924318194817;$

$d(\rho_{Cr-TR}(\tilde{r}_5, \tilde{r}^+), \rho_{ZL_1}) = 0.382830298614334.$

Rank all the alternatives $A_i (i = 1, 2, 3, 4, 5)$.

$A_1 < A_2 < A_4 < A_3 < A_5$.

Hence, the best alternative is A_1.

Table 3: Comparison of Ranking Methods in the literature

Methods	Rank of Alternatives
Existing Method	$A_1 < A_2 < A_4 < A_3 < A_5$ Best alternative is A_1.
Proposed Method [24]	$A_3 < A_2 < A_4 < A_5 < A_1$ Best alternative is A_3.
Proposed Method [26]	$A_1 > A_4 > A_2 > A_3 > A_5$ Best alternative is A_1.
Proposed Method [5]	$A_2 > A_3 > A_1 > A_4 > A_5$ Best alternative is A_2.

V11. CONCLUSION

In this paper, a new approach for finding the weights for decision making process based on partial differential equation solved through Sumudu Decomposition Method. The unknown decision maker weights are utilized in Decision making problem for identify the best alternatives. The feasibility of the proposed method is displayed through numerical example and comparison made with existing methods.

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